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Plastic design of steel frames for minimum weight

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PLASTIC DESIGN OF STEEL FRAMES

FOR MINIMUM WEIGHT

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A

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ABSTRACT

The purpose of this study was to develop a method for weight minimization of the plastically designed frame, braced normal to its plane of action, and composed of prismatic steel members. The method accounts for the non-linear relationship between weight and moment capacity for both beams and columns. Reduction in the pure-bending fully-plastic moment in the presence of axial loading and both beam-column instability and overall frame instability due to sidesway are taken into account. Provision is made for minimization of frames employing standard sections as well as for frames whose built-up members may be chosen from a continuous spectrum.

An initial solution to the minimization problem is obtained by the Simplex Method of linear programming, after which a check procedure is used to explore variations in the initial solution to determine if it can be improved. Simple portal frames with fixed end legs and hinged end legs are considered as a model. Design charts are established.

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NOTATION

a_i	=	Constant (Fig. 3)
a_{ij}	=	Coefficients in the array of linear restrictions. (Eq. 2-2)
a_K	=	Constant (Fig. 2)
b_i	=	Constant (Fig. 3)
b_K	=	Constant (Fig. 2)
c_i	=	Length of i^{th} column in ft.
c_j	=	Coefficients of variables in the objective function
c_K	=	Length of K^{th} beam in ft.
f	=	Objective function
F_W	=	Frame weight
F'_W	=	Frame weight minus a constant
G	=	Wind load factor
H	=	Column length
M_P	=	Pure-bending fully plastic moment
M_o	=	Moment capacity of a column of a given length in conjunction with a given axial load
m	=	Number of linear restrictions expressed by (Eq. 2-4)
n	=	Number of structural variables
n_b	=	Number of beams in frame
n_c	=	Number of columns in frame
P	=	Concentrated vertical load
P_y	=	Product of the cross sectional area and the yield stress of the steel
Q	=	Uniform loading on frame
R	=	Axial load on column (Fig 4.5)
r_x	=	Radius of gyration with respect to x-x axis
W_u	=	Uniformly distributed load per ft. of height (Eq. 3-1)
T	=	Lateral load (Fig. 4.5)
W_i	=	Weight per ft. of i^{th} column
W_K	=	Weight per ft. of K^{th} beam
X_1	=	Plastic moment for beam (Fig. 4.5)
X_2	=	Plastic moment for columns (Fig. 4.5)
X_j	=	Structural variables
X_{n+i}	=	Slack variables (Eq. 2-4)
X_{n+m+j}	=	Slack variables (Eq. 2-4)
θ	=	Mechanism angle (Fig. 4.5)

I. INTRODUCTION

1.1 General Remarks

The determination of the maximum load carrying capacity of a given frame is a problem for which only one answer exists; many feasible designs for a given geometry and loading may exist. Only one solution, however, provides the minimum cost design, or, as will be considered in this study, the minimum-weight design.

The indeterminate structure, when designed on the basis of elastic analysis, requires a method of trial to approach the minimum-weight design. This fact is manifested by the presence of the stiffness or flexibility factor in the matrix of coefficients which relate the redundant moments;it is usually found that different member sizes are required whereupon further analysis is necessary.

With the development of the plastic method of analysis however, there is now the possibility of the determination of an admissible distribution of moments over an indeterminate structure without an estimate of the member sizes having been made beforehand. This is because the matrix of coefficients which relate the critical moments are functions of the geometry of the structure and of the hinge positions, and are independent of member sizes.

Since member sizes do not have to be estimated before an analysis is made, one can determine a distribution of moments for a given structure which will yield a minimum-weight solution. In order that this may be accomplished, suitable relationships between weight of member and moment capacity of both beams and columns must be established. In addition, a method of proceeding efficiently from one distribution of moments to another which will yield a light weight structure must be developed.

1.2 Object and Scope

1. The object of this study is to develop a method for the minimum-weight design of steel structures, based on plastic analysis, which satisfies the following requirements:

   a. The method will embrace the problem of axial compression as well as flexural loading, lateral displacement (sidesway) of the structure, and the non-linear relationship between unit weight of members and their moment capacities.

   b. The method will allow the determination of minimum weight for the structure which is designed for standard structural shapes as well as the structure for which a continuous spectrum of shapes may be available.

2. Design charts are developed for simple portal frames.

1.3 Outline of Project

The problem considered in this study may be stated as follows: Given a set of static loads, acting at certain fixed points of a rigid-jointed plane frame of prescribed geometrical form, how should the cross sectional dimensions of the members be chosen to produce the lightest possible frame capable of carrying the loads? The members are required to be straight and of constant cross-section throughout their length.

To determine whether a frame will support the loads applied to it the theory of plastic collapse will be used. This theory applies to structures made of a ductile material, and assumes that if the curvature of a member becomes infinitely large the bending moment tends to a maximum value, called the fully plastic moment, which depends only on the section dimensions.

The problem is to minimize the weight of the frame subject to certain constraining conditions. These conditions are imposed by the fact that the moments in the structure must be in equilibrium with the known external loads, and must be less than or equal to the fully plastic moments of the members in which they occur. Mathematically, therefore, the problem is one of minimizing a linear function of several variables subject to a set of linear inequalities. This is the basic problem of linear programming.

This study develops an alternative system of analysis which provides an exact solution of the general problem, and which is particularly suitable for use on a digital computer. Two general cases of simple portal frames are analyzed and the design charts are plotted.

1.4 Review of Literature

Using concepts of plastic analysis, several researchers have made attempts to establish a minimum-weight design procedure for ultimate loading. J. Foulkes and J. Heyman (5)* have proposed a trial-and-error mechanism method that seems feasible only for simple structures. Foulkes (6) made a geometric interpretation of the work equations and succeeded in establishing a design chart for the simple case of a one story single-bay frame. Prager (21) later refined Foulkes' work by considering a nonlinear form of the weight-strength function and showed how the design chart for a portal frame was changed thereby. P.G. Hodge (12) investigated a method for minimum-weight design that does not depend on theoretical weight-strength functions, but works directly with the available sections. Because this is a trial-and-error solution, it would be too laborious for a complex frame.

In the case of multistory frames, the axial load effects are important. Their stability should be checked at the time of selection, as a normal design procedure. The effect of the axial forces and the stability of the beam-column to the minimum-weight problem will be considered in

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*The number in parentheses refers to bibliographical entries.

this study.

Since the majority of structures for which plastic analysis is presently considered appropriate are constructed of available standard structural shapes, a method which is to be useful to the designer must, in its final application, relate the minimum-weight problem to the properties of these shapes rather than to a continuous spectrum of shapes.

II. DEVELOPMENT OF MATHEMATICAL SOLUTION

2.1 General Remarks

The method of inequalities could be easily adapted to the problem of designing a frame for minimum weight if the weight per unit length of a structural member could be taken as linear and proportional to the fully plastic bending moment of this member. Under this assumption the minimum-weight design of a structural frame constitutes a problem in linear programming.

2.2 Assumptions and Limitations

a. This study is limited to frames in one plane, composed of rigidly jointed or pin-jointed members, which are braced normal to their plane of action.

b. Only prismatic steel members whose cross sections have an axis of symmetry lying in the plane of the structure are considered.

c. All loads act in the plane of the structure.

d. Lateral loads act only at the joints.

e. The distributed load is replaced by a set of equivalent concentrated loads.

f. Beam-columns are assumed to be subjected only to bending when the ratio of applied load to plastic axial load is less or equal to 0.15.

g. Further restrictions applying to particular problems will be discussed as they arise.

2.3 The Minimization Problem

For a given load system and structure geometry, many feasible designs may be determined. Mathematically this fact is expressed by the existence of more unknowns than there are equations which relate the unknowns. Furthermore, the distribution of moments over the structure at the ultimate load is influenced by the relative moment capacities of the various members. This fact becomes evident if the structure is observed during the last stages of loading leading to the ultimate load. As the ultimate load is approached, each succeeding hinge brings about a redistribution that would have prevailed had the structure remained elastic.

The desired method should proceed from the first solution to the minimum-weight solution with the consideration of only a very small percentage of the possible solutions. Furthermore, it should proceed from one solution to the next without having to restart the solution process. Finally, a criterion to identify the minimum-weight solution must be available.

Such a method exists. This method, known as linear programming, was first developed by George B. Dantzig, Marshall Wood, and their associates.

2.4 Linear Programming

This discussion of the linear programming method encompasses a description of the method without consideration of derivations or proof of theorems. For a rigorous treatment of the subject, a text (8) is available.

A function may be either maximized or minimized by the linear programming method. Since this study is concerned with minimization of weight, only minimization will be considered. The method described is known as the Simplex Method.

Let it be required to minimize

$$f = \sum_{j=1}^{n} C_j \ X_j \tag{2-1}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} X_j \geq b_i \qquad i = 1,2,\ldots. \ m \tag{2-2}$$

$$X_j \geq o \qquad j = 1,2,\ldots. \ n \tag{2-3}$$

Eq. (2-1) is known as the objective function and Eqs. (2-2) and (2-3) are the linear restrictions or side conditions. The Simplex Method requires both the objective function and the side conditions to be linear. In order to apply formal systematic solution procedures to this problem the above inequalities must be expressed as equalities as follows:

Minimize

$$f = \sum_{j=1}^{n} C_j \ X_j + \sum_{i=1}^{m} oX_{n+i} + \sum_{i=1}^{m} oX_{n+m+i} \tag{2-4}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} X_j - X_{n+i} = b_i \qquad i = 1,2,\cdots m \tag{2-5}$$

Where:

$$X_j \geq o$$
$$X_{n+i} \geq o$$

$X_{n+m+i} \geq 0$
X_j = Structural variables
X_{n+i} and X_{n+m+i} = Surplus variables
a_{ij}, b_i, C_j = Constant
n = Number of structural variables
m = Number of linear restrictions expressed by Eq: (2-5)

2.5 The Artificial-Base Technique

Instead of solving for an initial basic feasible solution, we may assume an entirely artificial one. To do this, we simply add to our augmented matrix an identity matrix consisting of the same number of new variables as we have equations. These new variables must be included in the objective function. However, we assign them such arbitrarily large coefficients as to drive them from the solution. The final solution is not valid unless all these artificial variables are absent. Further explanation and proof of validity of this technique may be found in Ref. (8).

2.6 The Objective Function

The objective function which is to be minimized in order to determine a minimum-weight steel frame is that function which expresses the total weight of the frame.

$$F_w = \sum_{K=1}^{n_b} C_K W_K + \sum_{i=1}^{n_c} C_i W_i \qquad (2\text{-}6)$$

where

F_w = Weight of frame
C_K = Length of K^{th} beam in ft.
W_K = Weight per ft. of K^{th} beam
n_b = Number of beam in frame
C_i = Length of i^{th} column in ft.
W_i = Weight per ft. of i^{th} column
n_c = Number of columns in frame

Although Eq. (2-6) is exact, it cannot be used in its present form because the side conditions provided by the mechanism equations relate the moment capacities of the individual members *and* not their weight. In order to effect compatibility between the objective function and the side conditions, the weights of the members must be expressed as functions of their moment capacities. These relationships will be considered separately for beams and columns.

In Fig. 1 the weight per ft., W, of wide-flange and other I-shapes, as given in Ref. (22), are shown plotted against their pure-bending plastic moment capacities M_P. The solid curve is drawn as a best fit of the "economy" sections. (If all available standard shapes are arranged in order of descending M_P, an economy shape is identified as that one lightest in weight which furnishes a value of M_P larger than those of the (heavier) shapes which intervene between it and the next lightest economy shape). In the investigation of design for minimum weight, only the economy sections need be considered. The curve of best fit for these sections has been determined in Ref. (15) to be:

$$W = 1.2\ M_P^{\frac{2}{3}} \qquad (2\text{-}7)$$

Since this equation is non-linear it cannot be substituted into the objective function. Fortunately, the range in M_P from the smallest shape that could be used to the largest shape which probably would be used for a particular loading is limited. For uniform load this range would normally be from $WL^2/16$ to $WL^2/8$. A number of trials showed that a straight line gives a good fit to the plot of economy sections for particular conditions of geometry and loaing and for a reasonable range of M_P. Line AB of Fig. 2 is typical. Therefore, we may write

$$W_K = a_K + b_K M_{PK} \qquad (2\text{-}8)$$

where M_{PK} = Plastic moment capacity of the K^{th} beam.

As in the case of beams, only the economy sections need be considered for columns for the minimum-weight problem. The range of M_P for the column will normally extend from the smallest shape that could be used to the maximum M_P, the column would receive from adjacent beams. Line C-D, Fig. 3, is a typical best-fit straight line. The equation is

$$W_i = a_i + b_i M_{Pi} \qquad (2\text{-}9)$$

where M_{Pi} = Plastic moment capacity of the i^{th} column.

The objective function, Eq. (2-6) is now expressed as

$$F_W = \sum_{K=1}^{n_b} (C_K a_K + C_K b_K M_{PK}) + \sum_{i=1}^{n_c} (C_i a_i + C_i b_i M_{Pi}) \qquad (2\text{-}10)$$

when substitutions are made for W_K and W_i. Because all terms $C_K a_K$ and $C_i a_i$ are constants for a particular problem, these terms may be dropped in determining the values of M_{PK} and M_{Pi} which yields minimum frame weight, i.e., the value of F_W is not sought, but rather the values of M_{PK} and M_{Pi} which yields the minimum F_W. Therefore, the objective

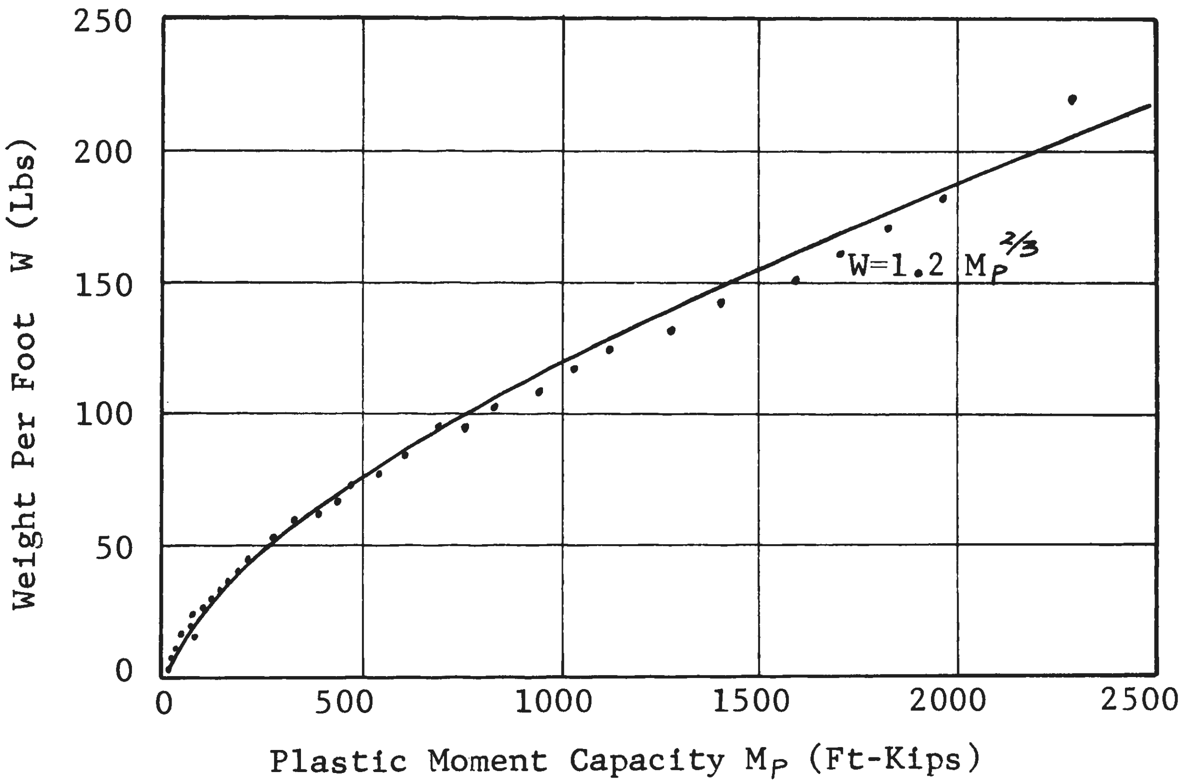


FIG. 1 WEIGHT PER FOOT VS PLASTIC MOMENT CAPACITY OF ECONOMY SECTIONS

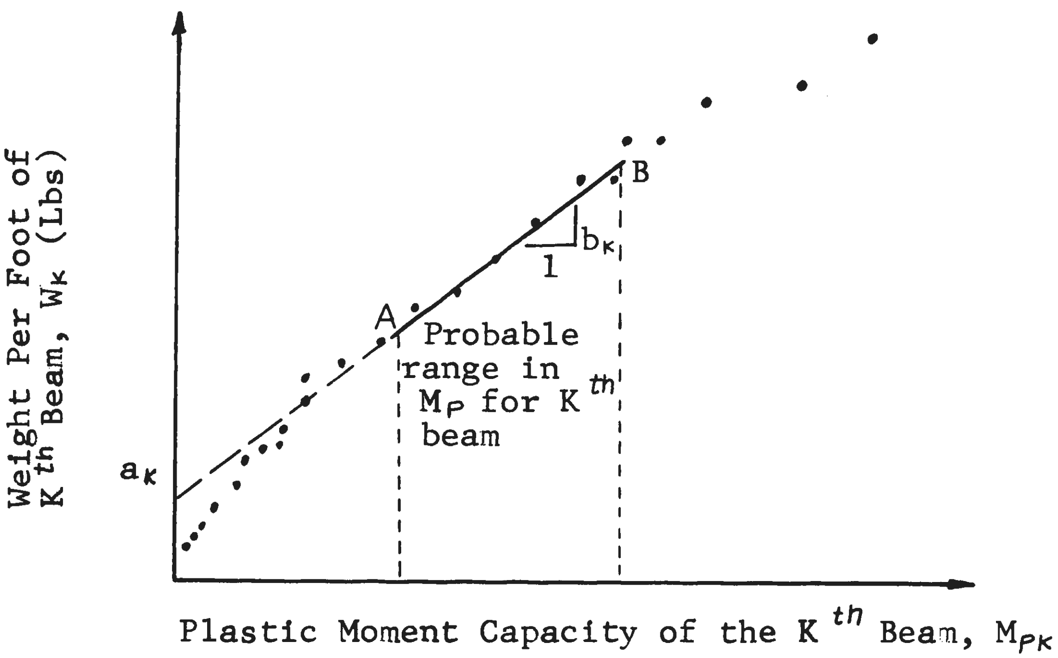


FIG. 2 RELATIONSHIP BETWEEN W_K AND M_{PK} FOR BEAMS

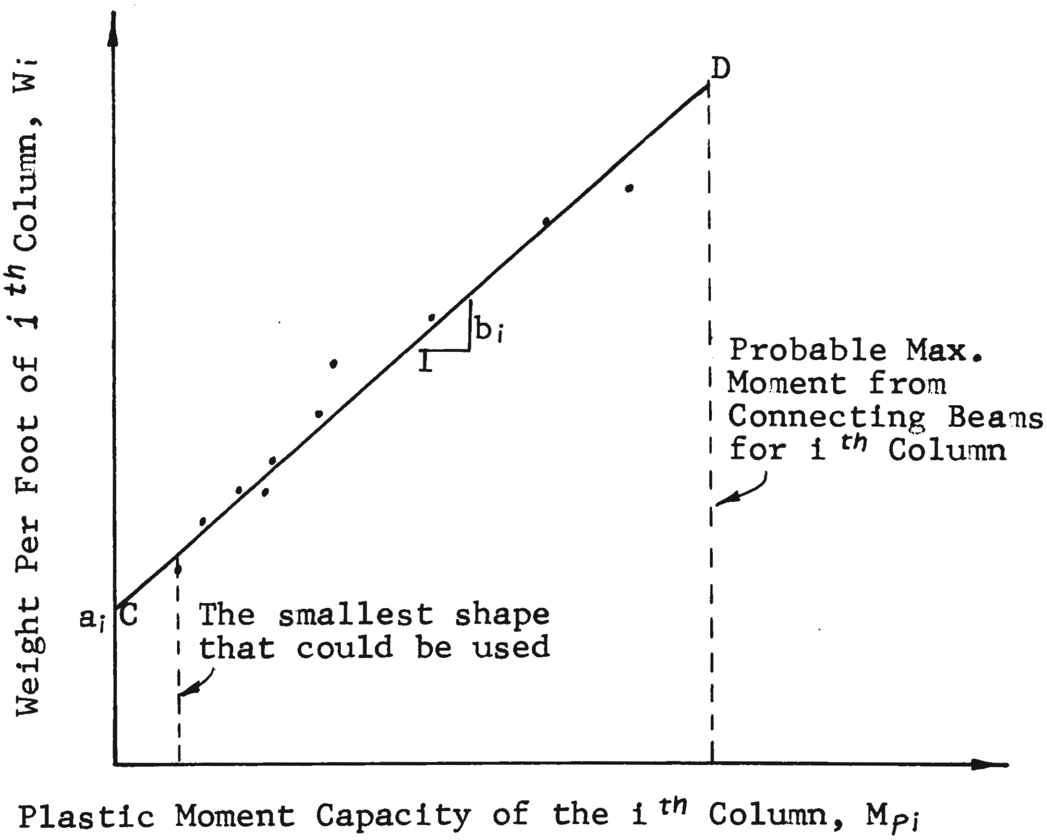


FIG. 3 RELATIONSHIP BETWEEN W_i AND M_{pi} FOR BEAM COLUMNS

function may be written:

$$F_W' = \sum_{K=1}^{n_b} C_K b_K M_{PK} + \sum_{i=1}^{n_c} C_i b_i M_{Pi} \qquad (2\text{-}10)$$

where F_W' = Frame weight minus a constant.

In order to more conveniently express the objective function in the linear programming tableau, Eq. (2-10) will be written:

$$F_W' = \sum_{j=1}^{n} C_j X_j \qquad (2\text{-}11)$$

where

$C_j = C_K b_K$ if j^{th} member is a beam.
$C_j = C_i b_i$ if j^{th} member is a column.
$X_j = M_{PK}$ if j^{th} member is a beam.
$X_j = M_{Pi}$ if j^{th} member is a column.

2.7 Linear Restrictions

A system of infinitesimal displacements, made possible by the insertion of an adequate number of yield hinges into the otherwise rigid members of the structure, specifies a flow mechanism. Given loads are beyond the load-carrying capacity of a beam or frame if a flow mechanism exists for which the work of these loads exceeds the energy dissipated in the plastic bending at the yield hinges. Conversely, the absence of such a flow mechanism indicates that the given loads are within the load-carrying capacity of the structure. A flow mechanism for which the energy dissipated in the yield hinges equals the work of the given load will be termed a failure mechanism for these loads.

The linear restrictions or side conditions originate from the mechanism inequalities and possibly, from other design requirements. Further restrictions could arise in the form of arbitrary limitations set by the designer. For example, it may be desirable to limit either the maximum or minimum moment capacity, or both, of one or more members, in which case additional inequalities are required.

To insure compliance with the criteria of yield and equilibrium, inequalities representing all possible modes of failure should be included in the formulation of the problem.

2.8 Sidesway Effect

Failure of a frame may result from overall instability involving sidesway at an ultimate load less than that which would be carried if the frame were braced to prevent sidesway. At the present time the ultimate load with respect to this form of instability cannot be predicted precisely. However, in Ref. (3) the following expression is suggested for columns subject to sidesway.

$$\frac{2P}{P_y} + \frac{L}{70r_x} \leq 1 \qquad (2\text{-}12)$$

where

P = Applied load (Kips).
P_y = Plastic axial load; equal to profile area times specified minimum yield point (Kips).
L = Actual unbraced length (inches).
r_x = Radius of gyration with respect to the plane of bending (inches).

This equation is conservative for frames of proportions likely to be found in practice. It has been adopted by the A.I.S.C. Specification Committee as an interim provision.

2.9 Axial Load Effect

In addition to causing column instability the presence of axial force tends to reduce the magnitude of the plastic moment. Therefore, the column sizes should be checked, at the time of selection, as a normal design procedure. The effect is small in the case of small axial loads, and therefore in ordinary portal frame columns any reduction in hinge moment usually may be ignored. However, in the case of multistory structures, the resisting moment of the columns in the lower stories would be reduced by axial load and evaluation of the ultimate load must then include such considerations.

Changes in size of members in proceeding from one basic feasible design to another affect the distribution of moments over the structure. To a lesser extent, the distribution of axial load is also affected. However, it will be assumed here, as is usually done, that axial load remain constant with change in member size.

Galambos and Ketter (7) developed interaction formulas relating moment capacity M_o and axial compression P for the following three cases:

Case I. For columns bent in double curvature by moments producing plastic hinges at both ends of the columns

$$M_o = M_p \quad \text{when } P/P_y \leq 0.15 \qquad (2\text{-}13)$$

$$\frac{M_o}{M_p} \leq 1.18 - 1.18 \left(\frac{P}{P_y}\right) \leq 1.0 \quad \text{when } P/P_y > 0.15$$

Case II. For pin-based columns required to develop a hinge at one end only, and double curvature columns required to develop a hinge at one end when the moment at the other end would be less than the hinge value.

$$\frac{M_o}{M_p} \leq B - G\left(\frac{P}{P_y}\right) \leq 1.0 \qquad (2\text{-}14)$$

where

$$B = 1.13 + \frac{L/r}{3080} + \frac{(L/r)^2}{185000}$$

$$G = 1.11 + \frac{(L/r)}{190} - \frac{(L/r)^2}{9000} + \frac{(L/r)^3}{720{,}000}$$

$$M_o = M_p \quad \text{When } P/P_y \leq 0.15$$

Case III. For columns bent in single curvature

$$\frac{M_o}{M_p} \leq 1.0 - H\left(\frac{P}{P_y}\right) - J\left(\frac{P}{P_y}\right)^2 \qquad (2\text{-}15)$$

where

$$H = 0.42 + \frac{(L/r)}{70} - \frac{(L/r)^2}{29000} + \frac{(L/r)^3}{1160000}$$

$$J = 0.77 - \frac{(L/r)}{60} + \frac{(L/r)^2}{8700} - \frac{(L/r)^3}{606000}$$

The Specification of the American Institute of Steel Construction, which is the most widely accepted code for plastic design of steel frames, incorporates the Galambos-Ketter formula.

2.10 Description of Proposed Design Procedure

The proposed design procedure which was developed to determine the minimum-weight design of frames is described here briefly.

The solution is accomplished in the following steps:

1. The range in M_p to be considered for each beam and column.
2. Calculate the slope of the best fit straight line by the method of least squares for weight per foot vs. M_p equations for both beams and columns.
3. Calculate objective function coefficients as products of member length and slopes of best fit straight lines.
4. Determine linear restrictions.
5. The inequalities are augmented by the slack and artificial variables to provide an array of equalities.
6. The solution is effected by the Simplex Method, yielding the theoretical minimum-weight moments.
7. Using these moments, the lightest section is selectd for each member from the list of the standard economy sections. This becomes the initial solution.
8. Determine critical mechanisms and find maximum axial load for columns.
9. Check sidesway requirements of columns.
10. Check reduction of the plastic moment due to axial load.

III. DEVELOPMENT OF MODEL

3.1 General Remarks

Rectangular portal frames with pinned bases and fixed bases are considered as the model. Design charts are developed that give, at a glance, the minimum-weight design for various geometries and loading conditions of a portal frame. The distributed load is replaced by a set of equivalent concentrated loads. The wind load is indicated as horizontal load T concentrated at the eaves.

3.2 Portal Frame With Hinged Legs

Single-bay, one-story bents may be considered as beams that have been bent to the shape of a frame. Consider the portal shown in Fig. 4a, all the members of which are capable of carrying bending and shear as well as axial force. The legs are hinged at their bases and rigidly connected to the cross girder at the top. This structure is statically indeterminate to the first degree. The uniform loading is replaced by a set of equivalent concentrated loads and the wind load is indicated as horizontal load T concentrated at the eaves as shown in Fig. 4b.

For positive values of load T and P, only the mechanisms c through l (Fig. 4) must be fulfilled if the given loads are not to exceed the load-carrying capacity of the frame. The axial load for each mechanism is also shown in Fig. 4.

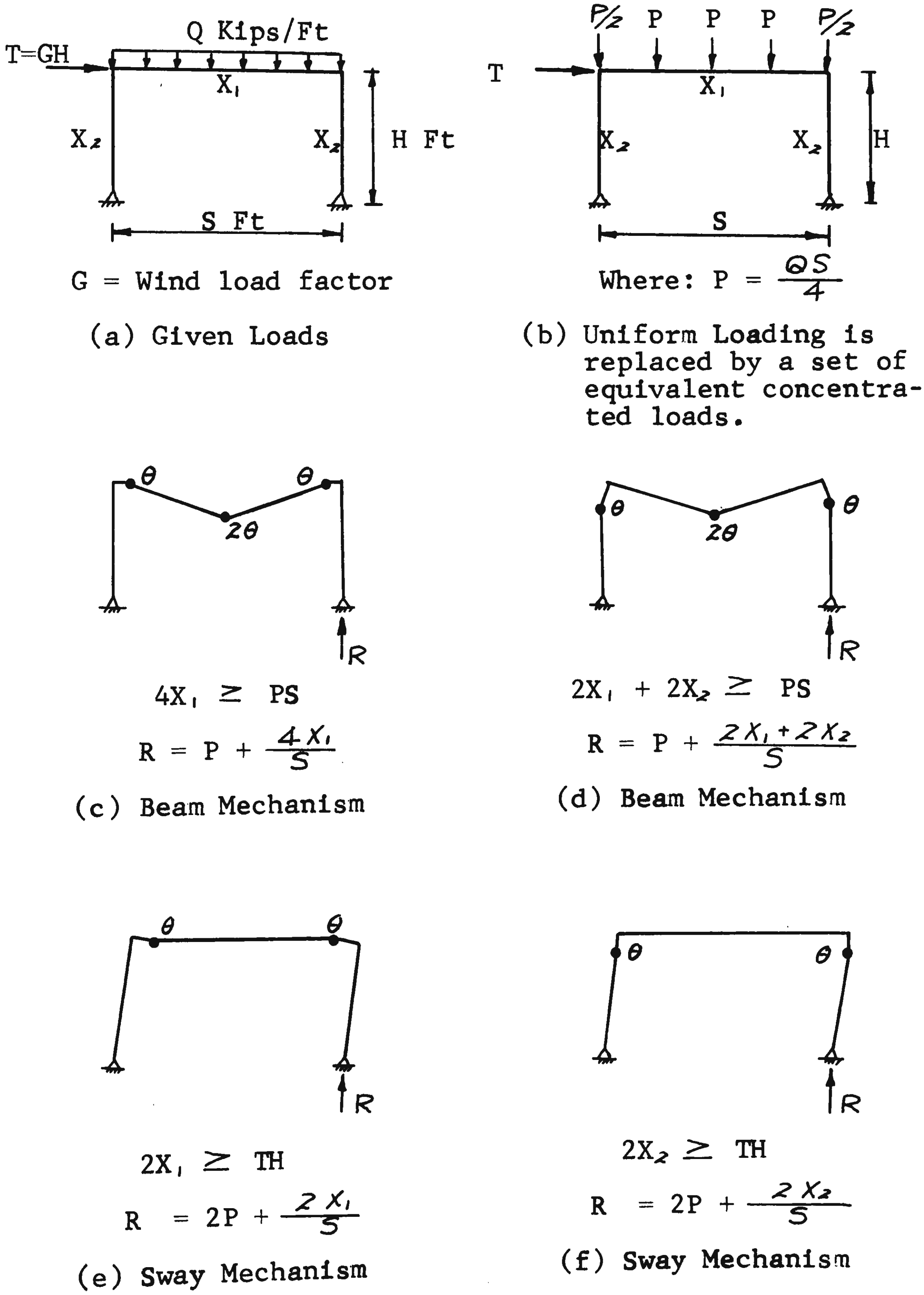
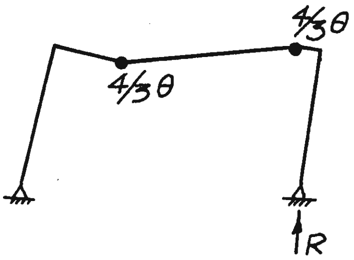


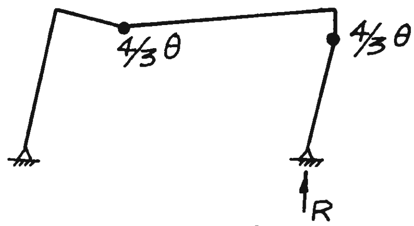
FIG. 4 MECHANISMS FOR A PORTAL FRAME WITH HINGED LEGS



$$8X_1 \geq \frac{3PS}{2} + 3TH$$

$$R = \frac{3P}{2} + \frac{8X_1}{3S}$$

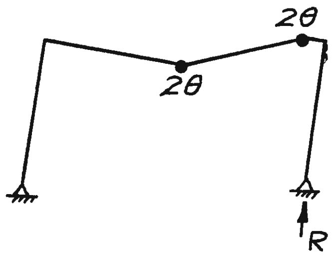
(g) Composite Mechanism



$$4X_1 + 4X_2 \geq \frac{3PS}{2} + 3TH$$

$$R = \frac{3P}{2} + \frac{4(X_1 + X_2)}{3S}$$

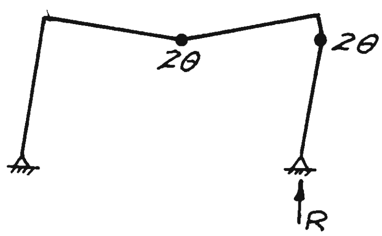
(h) Composite Mechanism



$$4X_1 \geq PS + TH$$

$$R = P + \frac{4X_1}{S}$$

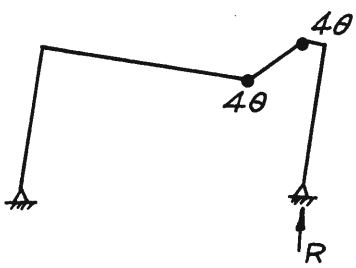
(i) Composite Mechanism



$$2X_1 + 2X_2 \geq PS + TH$$

$$R = P + \frac{2(X_1 + X_2)}{S}$$

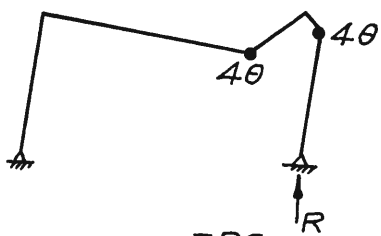
(j) Composite Mechanism



$$8X_1 \geq \frac{3PS}{2} + TH$$

$$R = \frac{P}{2} + \frac{8X_1}{S}$$

(k) Composite Mechanism



$$4X_1 + 4X_2 \geq \frac{3PS}{2} + TH$$

$$R = \frac{P}{2} + \frac{4(X_1 + X_2)}{S}$$

(l) Composite Mechanism

FIG. 4 MECHANISMS FOR A PORTAL FRAME WITH HINGED LEGS (continued)

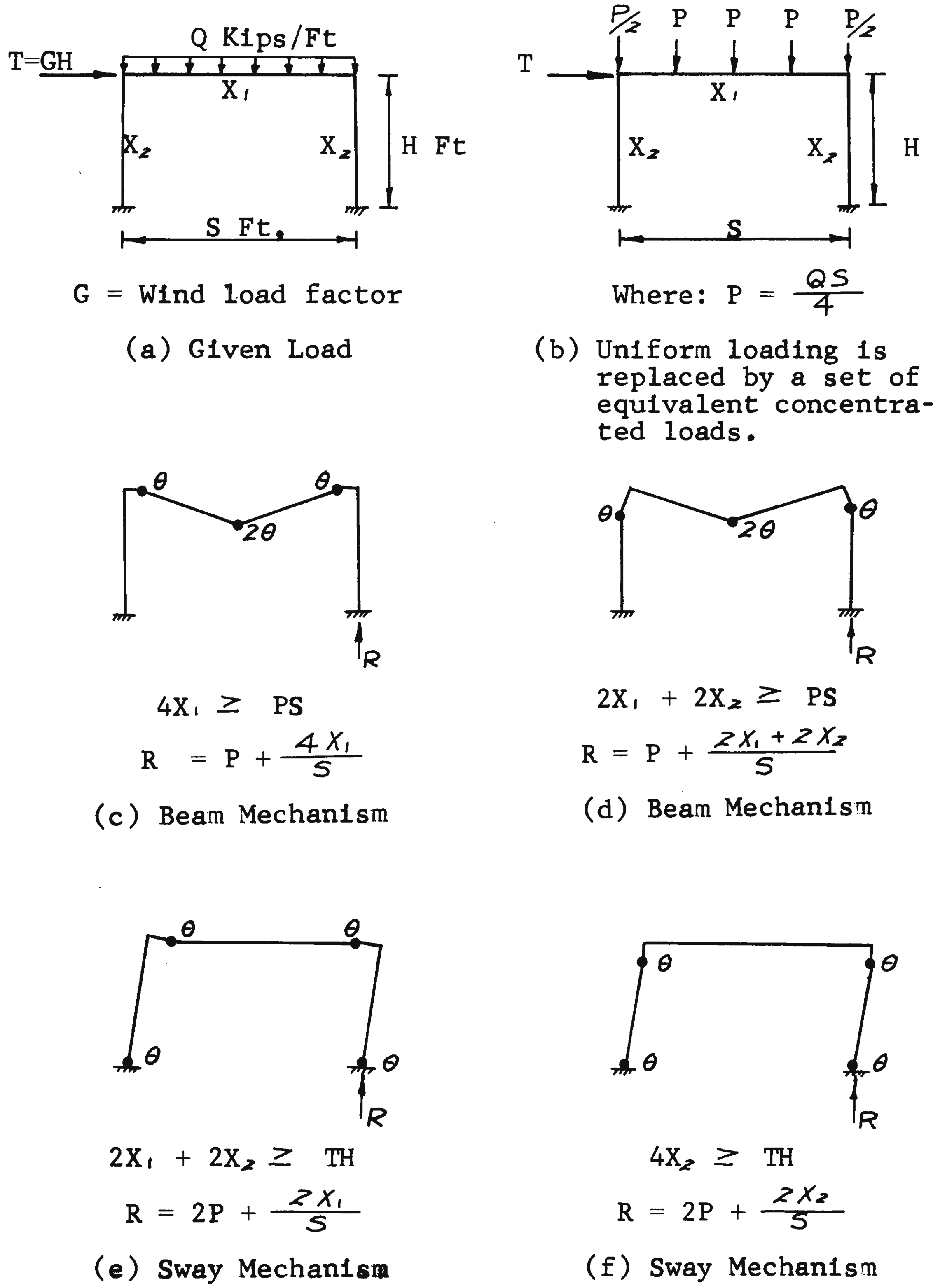
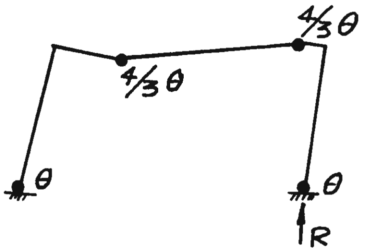


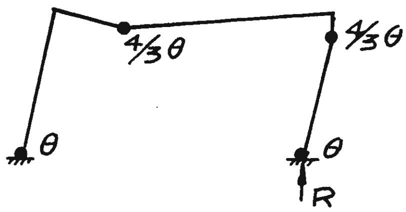
FIG. 5 MECHANISMS FOR A PORTAL FRAME WITH FIXED LEGS



$$\frac{8}{3}X_1 + 2X_2 \geq \frac{PS}{2} + TH$$

$$R = \frac{3P}{2} + \frac{8X_1}{3S}$$

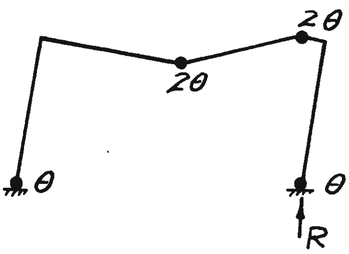
(g) Composite Mechanism



$$4X_1 + 10X_2 \geq \frac{3PS}{2} + 3TH$$

$$R = \frac{3P}{2} + \frac{4(X_1 + X_2)}{3S}$$

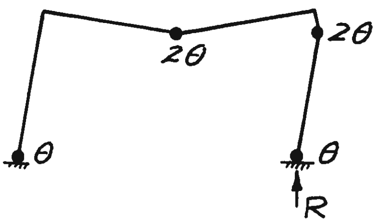
(h) Composite Mechanism



$$4X_1 + 2X_2 \geq PS + TH$$

$$R = P + \frac{4X_1}{S}$$

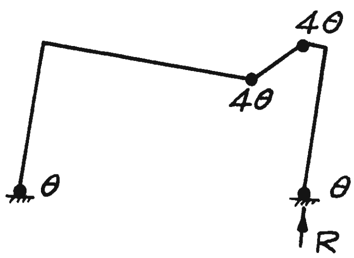
(i) Composite Mechanism



$$2X_1 + 4X_2 \geq PS + TH$$

$$R = P + \frac{2(X_1 + X_2)}{S}$$

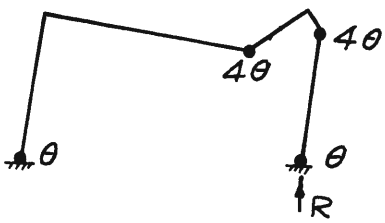
(j) Composite Mechanism



$$8X_1 + 2X_2 \geq \frac{3}{2}PS + TH$$

$$R = \frac{P}{2} + \frac{8X_1}{S}$$

(k) Composite Mechanism



$$4X_1 + 6X_2 \geq \frac{3}{2}PS + TH$$

$$R = \frac{P}{2} + \frac{4(X_1 + X_2)}{S}$$

(l) Composite Mechanism

FIG. 5 MECHANISMS FOR A PORTAL FRAME WITH FIXED LEGS (continued)

3.3 Portal Frame With Fixed Legs

Plastic analysis and design of the hingeless bent involves no more work than the two-hinged one. This is in welcome contrast to the situation encountered in the elastic design of rigid bents.

The two types of bent differ in that, instead of free hinges at the bottoms of the columns, there are potential plastic ones in the hingeless bent. The latter condition is brought out in the sway mechanisms of Fig. 5 where horizontal wind loads play an important role.

Consider now a portal similar in some ways to that of Fig. 4a but with the bases of legs fixed, as shown in Fig. 5a. For positive values of load T and P, only the mechanisms c through l (Fig. 5) must be fulfilled if the given loads are not to exceed the load-carrying capacity of the frame. The axial load for each mechanism is also shown.

3.4 Uniform Loading

Plastic hinges form at joints in the structure and at maximum moment points. With uniform loading the location of the maximum point is not always readily apparent. In such cases, the location of the plastic hinge is denoted by the parameter x and the virtual work equation for M_p written in terms thereof. This equation is then maximized to find x.

With errors that are usually slight, the analysis could be made on the basis that the uniform loading is replaced by a set of equivalent concentrated loads. Thus in Fig. 6, if the distributed load WL = P is concentrated in the various ways shown, the uniform load parabola is always circumscribed (giving the same maximum shear). The result is always conservative because the actual moment in the beam is always less than or at most equal to the assumed moment. Of course, the more concentrated loads assumed, the closer is the approximation to the real problem.

If the distributed load is actually brought to the main frame through purlins and girts, the uniform load may be converted, at the outset, to actual purlin reactions (on the basis of assumed purlin spacing). The analysis is then made on the basis of the actual concentrated loads. The only difficulty with this procedure is that numberous additional possible plastic hinges are created - one at each purlin. And for every possible hinge position there is another possible mechanism.

The total uniformly distributed load WL may be divided into any desired number of equal parts and spaced at equal distances from each other, so long as the end loads are each one-half the uniform spacing from the end. The greater the number into which WL is divided the more nearly the uniform moment diagram is approached.

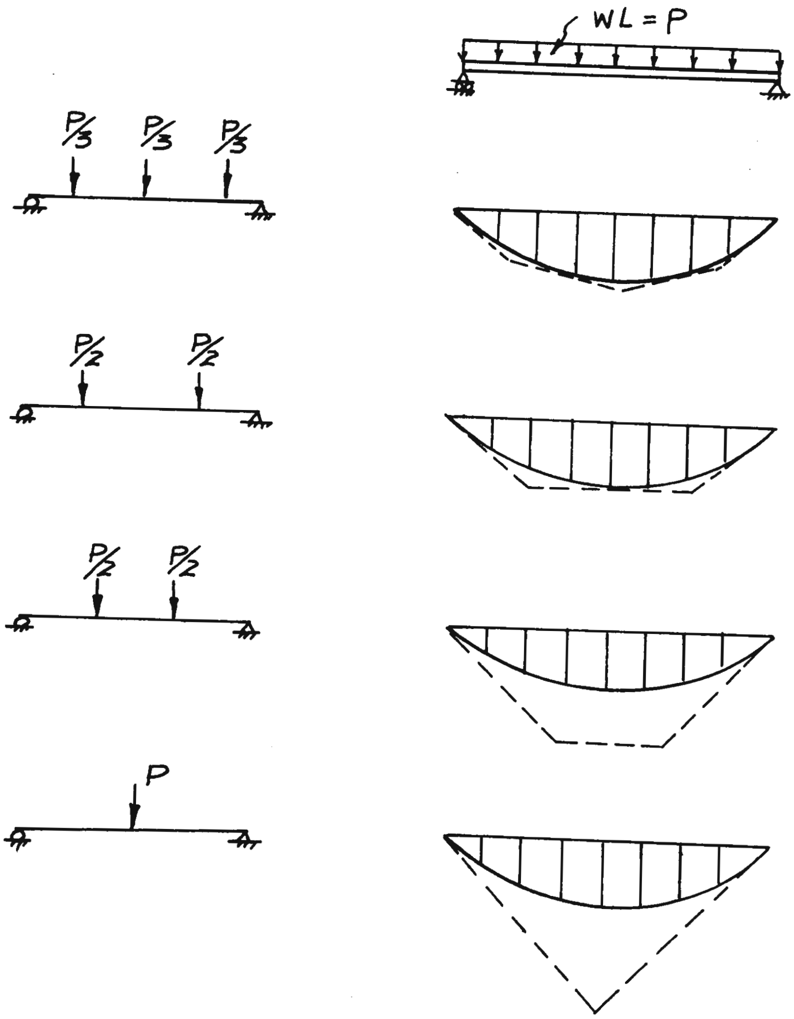


Fig. 6 The Effect of Replacing a Uniform Loading by an Equivalent Set of Concentrated Loads

3.5 Wind Loads

As will be noted from Figs. 4a and 5a, the wind load is indicated as horizontal load T concentrated at the eaves. Specifications invariably call for a given uniformly distributed load in pounds per square foot on a vertical surface. The load T must be of an amount such that its overturning moment about the base of the column is the same as that of the specified uniformly applied load. Let W_u be the uniformly distributed load per ft. of height. Then,

$$T = \frac{W_u H^2}{2H} = \frac{W_u H}{2} \tag{3-1}$$

A value for the velocity pressure q, under average conditions is given as:

$$q = 0.0026 \quad V^2 \tag{3-2}$$

in which q is the velocity pressure in pounds per square foot on a vertical surface and V is the wind velocity in miles per hour. In its Fifth Progress Report ASCE Sub-Committee 31 indicated that 0.8 q be taken as pressure on the windward side and 0.5 q as suction on the leeward side of the building; and, in its final report, it recommended that pressure on the windward side and suction on the leeward side be kept separate in the case of drill halls, hangars, industrial buildings, and other one-story buildings with spacious interiors.

In keeping with the final report of the Sub-Committee, it is recommended that 15 psf pressure be used on the vertical portion of the windward side of one-story bents and 9.5 psf suction be applied to the vertical portion of the leeward side of the building.

3.6 Description of Computer Program

The computer program which was developed to determine the minimum-weight design of frames is described here briefly. The flow chart is shown in Appendix D.

The input consists of the following data:

1. The nominal depth and weight per foot, the plastic moment capacity M_p, the radius of gyration r_x of the standard "economy" sections (Appendix B)
2. The array of mechanism inequalities.
3. The lengths of all members.
4. The range in M_p to be considered for each beam and column.
5. The condition of the frame (braced or unbraced) with respect to sidesway.

The output consists of the following information:

1. Maximum axial load for each column.
2. The equation of the corresponding mechanism of collapse
3. The theoretical moments and the initial design.
4. The least-weight design.
5. The frame weight per unit span length.

IV. RESULTS

4.1 General Remarks

The results in this section are based on steel whose yield stress is 33 ksi (A-7 steel). As has been pointed out in preceeding sections, provisions of the American Institute of Steel Construction Specifications for plastic design have been adhered to in respect to column stability, reduction in plastic moment capacity in the presence of axial force (columns only), lateral (sidesway) instability, etc. The computer program described in Sec. 3.6 was used to get the results, using the IBM 1620.

4.2 Results

Two cases of simple portal frame, one with fixed end legs and the other with hinged end legs under uniform loading and wind loading are considered.

(a) Assumed Data:

1. Frames are 20 feet on center
2. Load factor = 1.4
3. Wind load = 18 and 36 $Lb./Ft^2$

(b) Many different designs are investigated by:

1. Varying the intensity of distributed load, Q = 0.25, 0.5, 1, 2, 4, 8 Kips/Ft.
2. Varying the height of frame, H = 10, 15, 20 Ft.
3. Varying the wind loading, G = 0.25 and 0.5 Kips/Ft.
4. Studing the optimum span length of frame for varying end conditions, height of column and

loading.

For frames with fixed end legs, the variation of the plastic moment for beams and columns as a function of the loading and span length is shown in Fig. 7 and Fig. 8. Variation of the axial load on the column resulting from an increase in the loading and span length is shown in Fig. 10. The frame weight per unit span length for varying end conditions, height of column and loading are shown in Appendix B.

The sensitivity of the optimum span length of frame caused by a variation in wind loading, uniform loading, and the effect of height of frame is shown in Fig. 11 through 14.

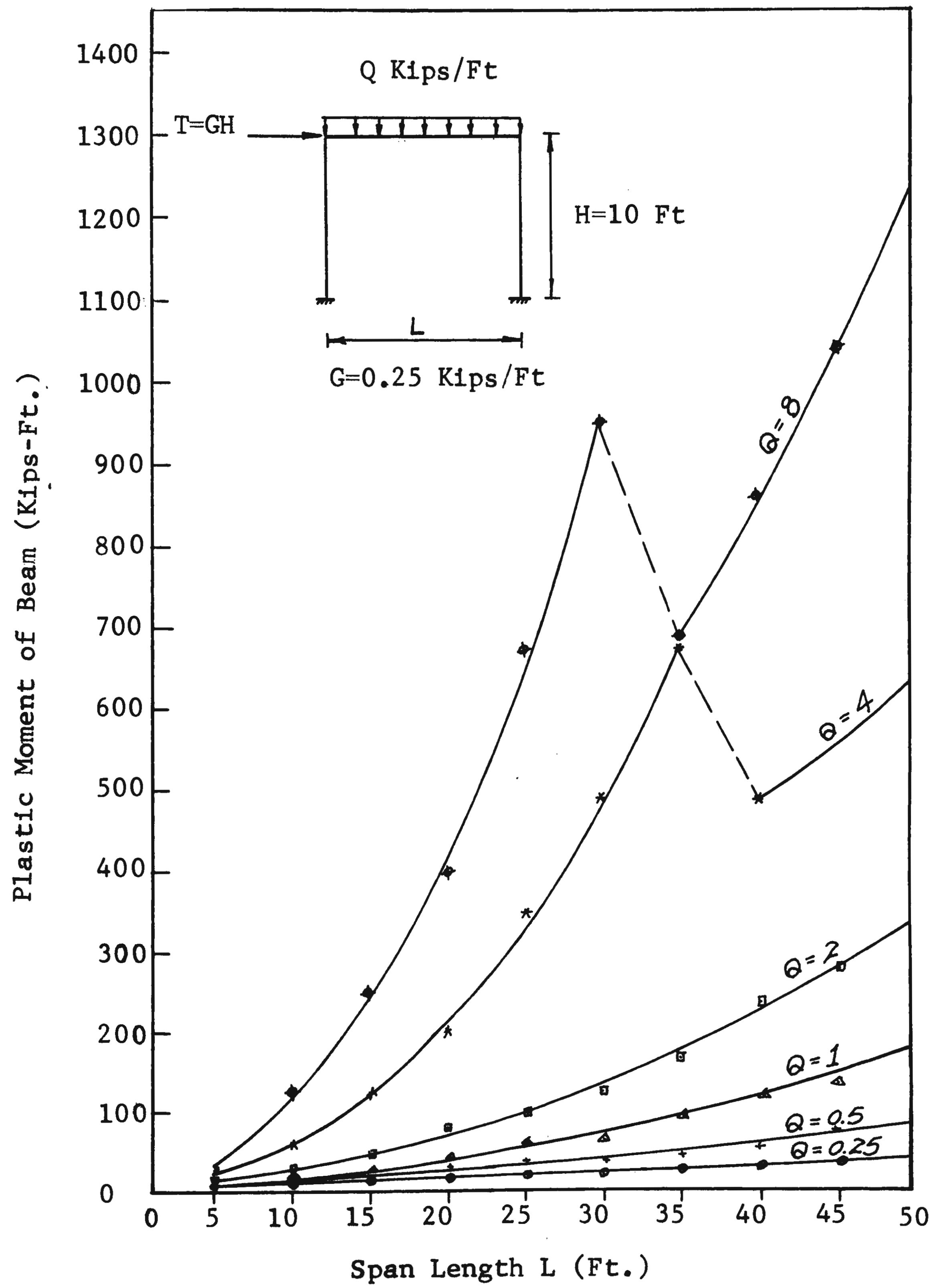


FIG. 7 PLASTIC MOMENT OF BEAM VERSUS SPAN LENGTH FOR A FIXED END PORTAL FRAME

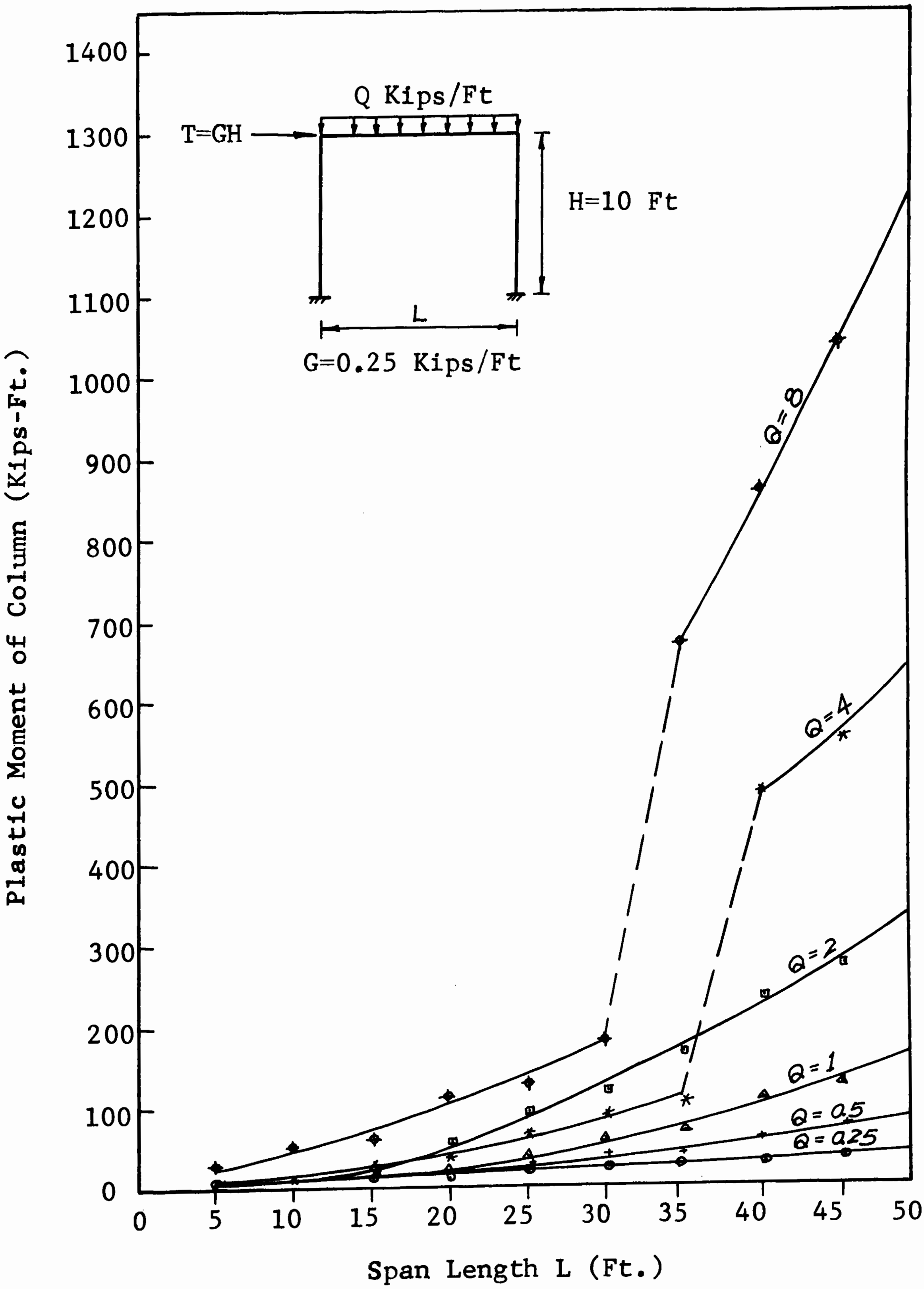


FIG. 8 PLASTIC MOMENT OF COLUMN VERSUS SPAN LENGTH FOR A FIXED END PORTAL FRAME

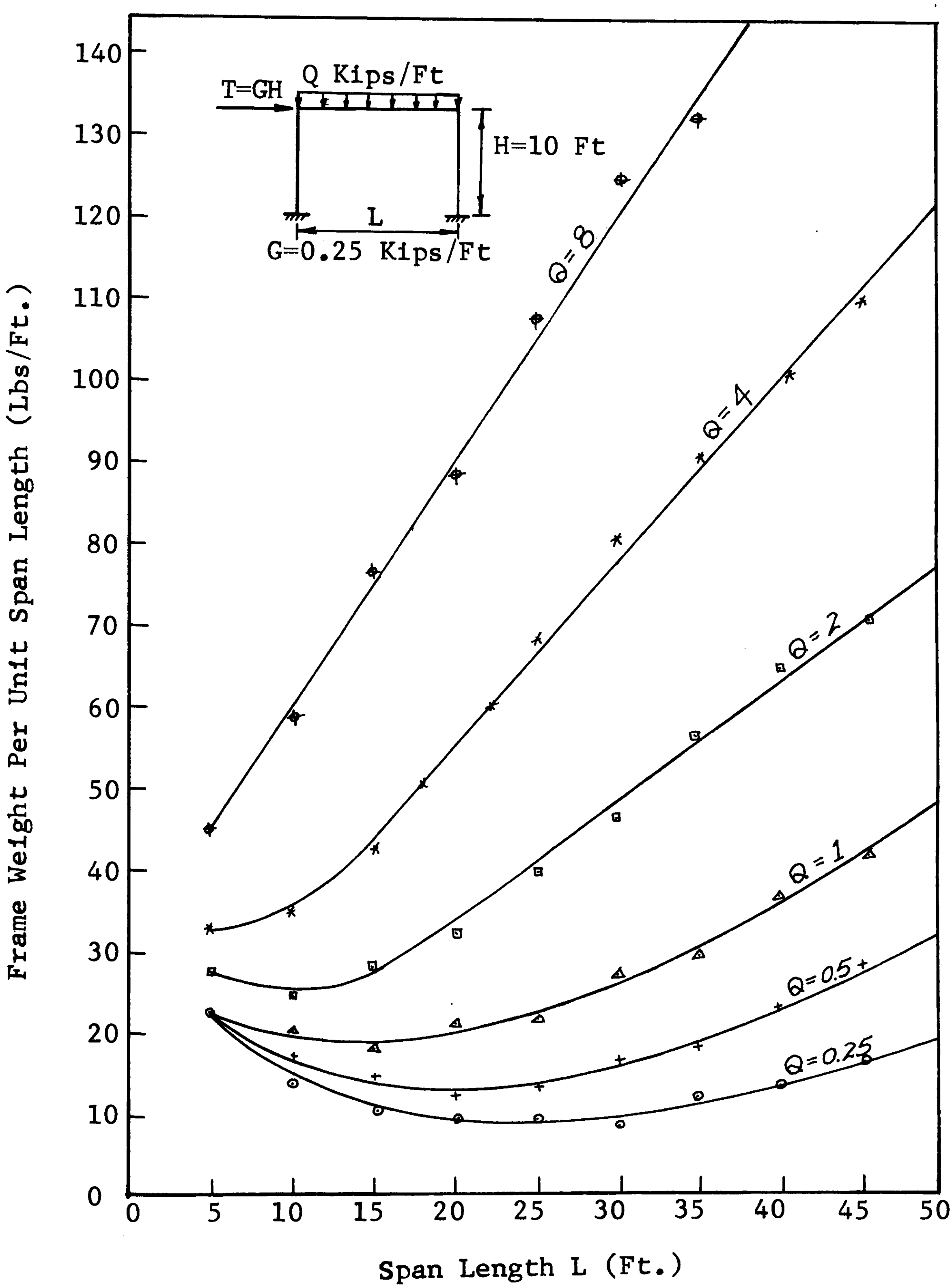


FIG. 9 FRAME WEIGHT VERSUS SPAN LENGTH FOR A FIXED-BASED PORTAL FRAME

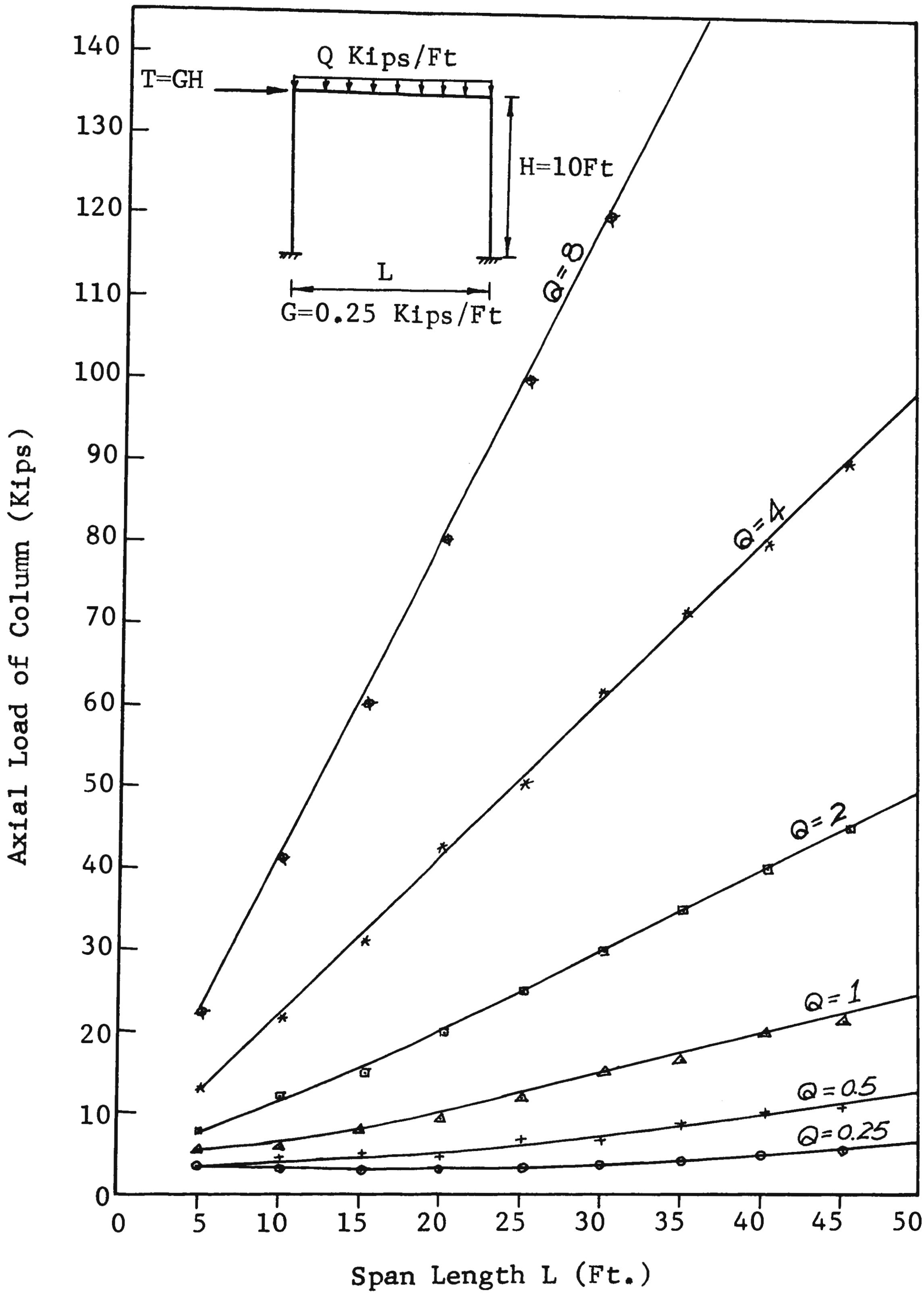


FIG. 10 AXIAL LOAD OF COLUMN VERSUS SPAN LENGTH FOR A FIXED END PORTAL FRAME

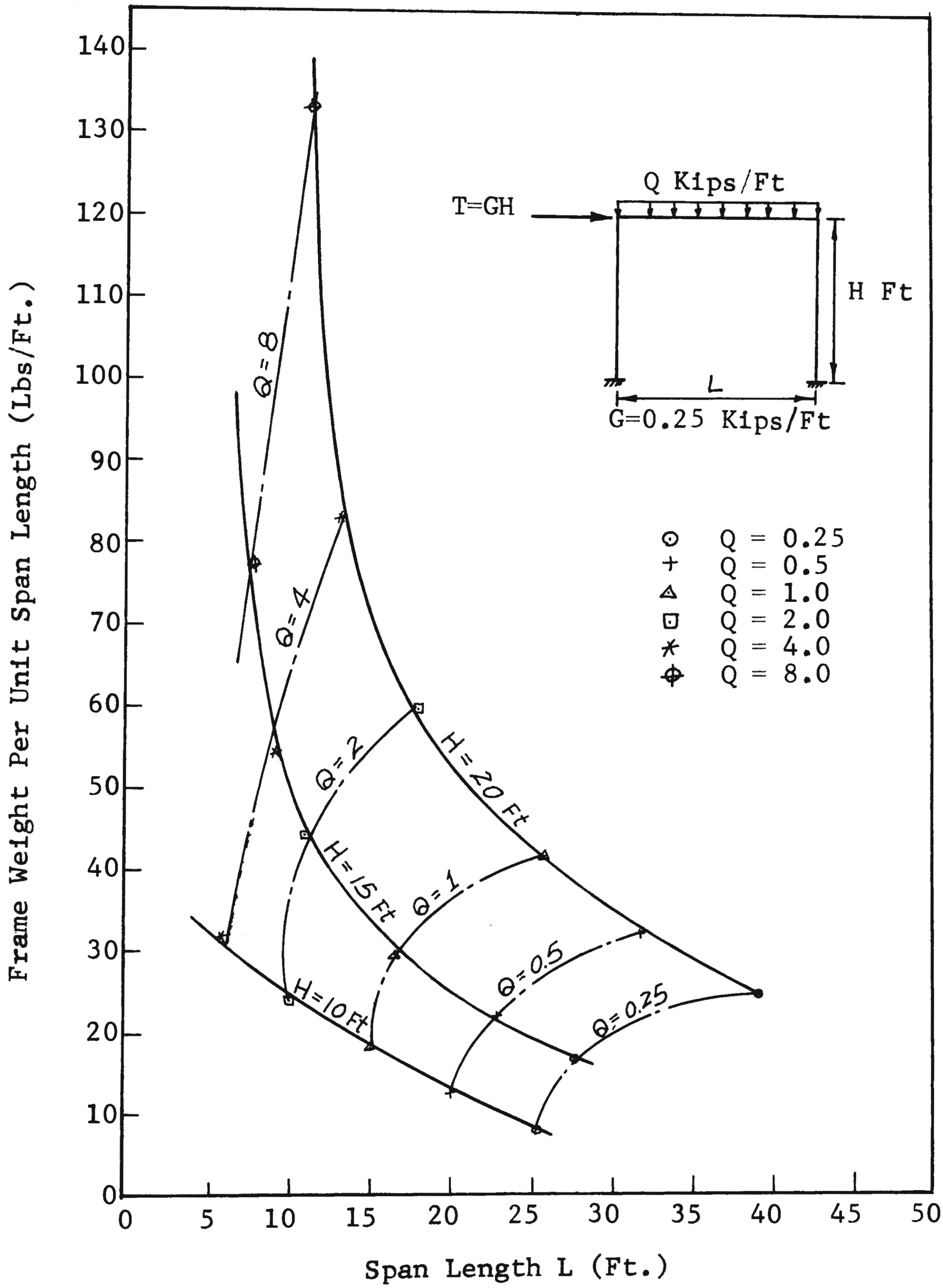


FIG. 11 FRAME WEIGHT VERSUS OPTIMUM SPAN LENGTH FOR FIXED END PORTAL FRAMES WITH WIND LOAD OF G = 0.25

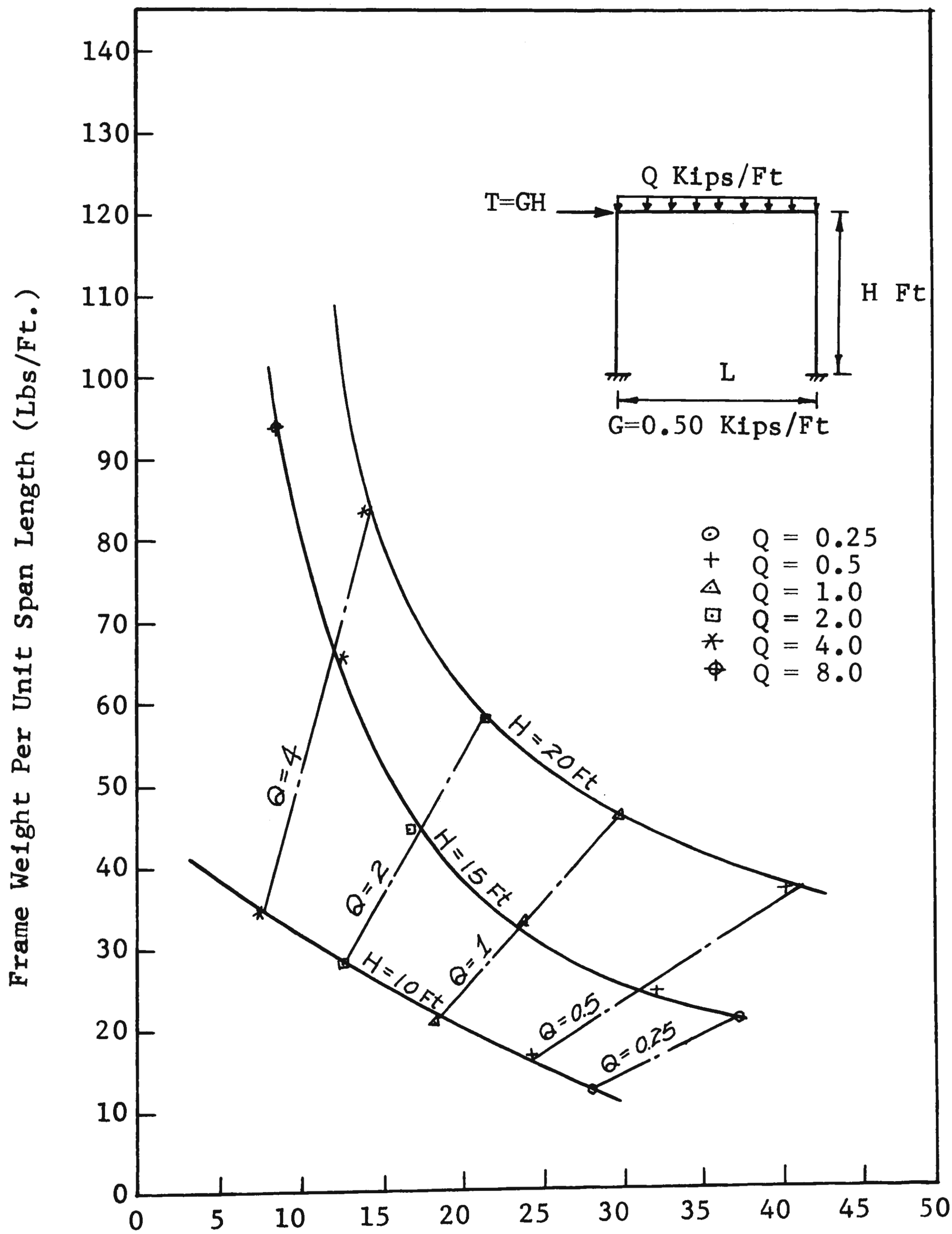


FIG. 12 FRAME WEIGHT VERSUS OPTIMUM SPAN LENGTH FOR FIXED END PORTAL FRAMES WITH WIND LOAD G = 0.50

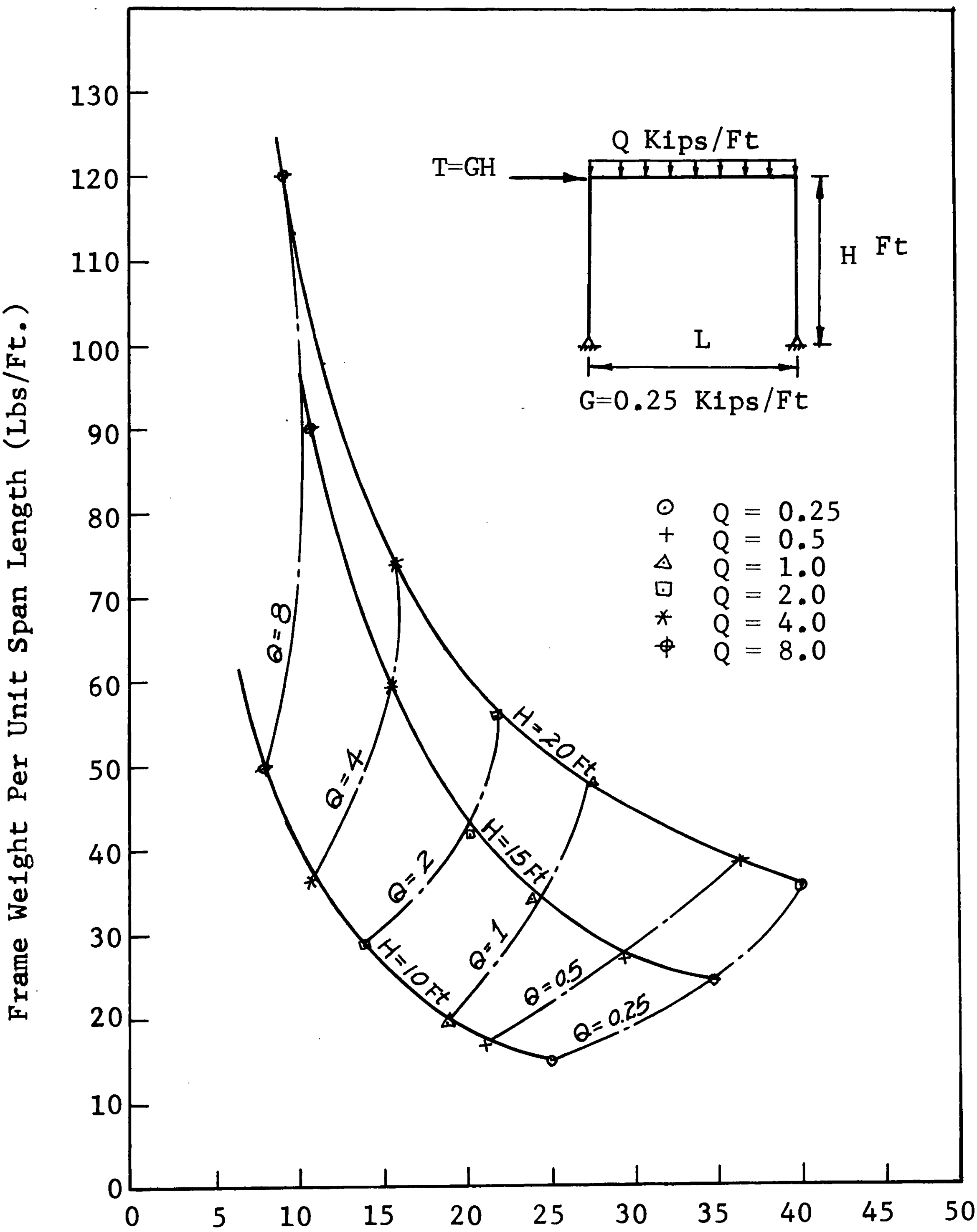


FIG. 13 FRAME WEIGHT VERSUS OPTIMUM SPAN LENGTH FOR HINGED END PORTAL FRAMES WITH WIND LOAD OF G = 0.25

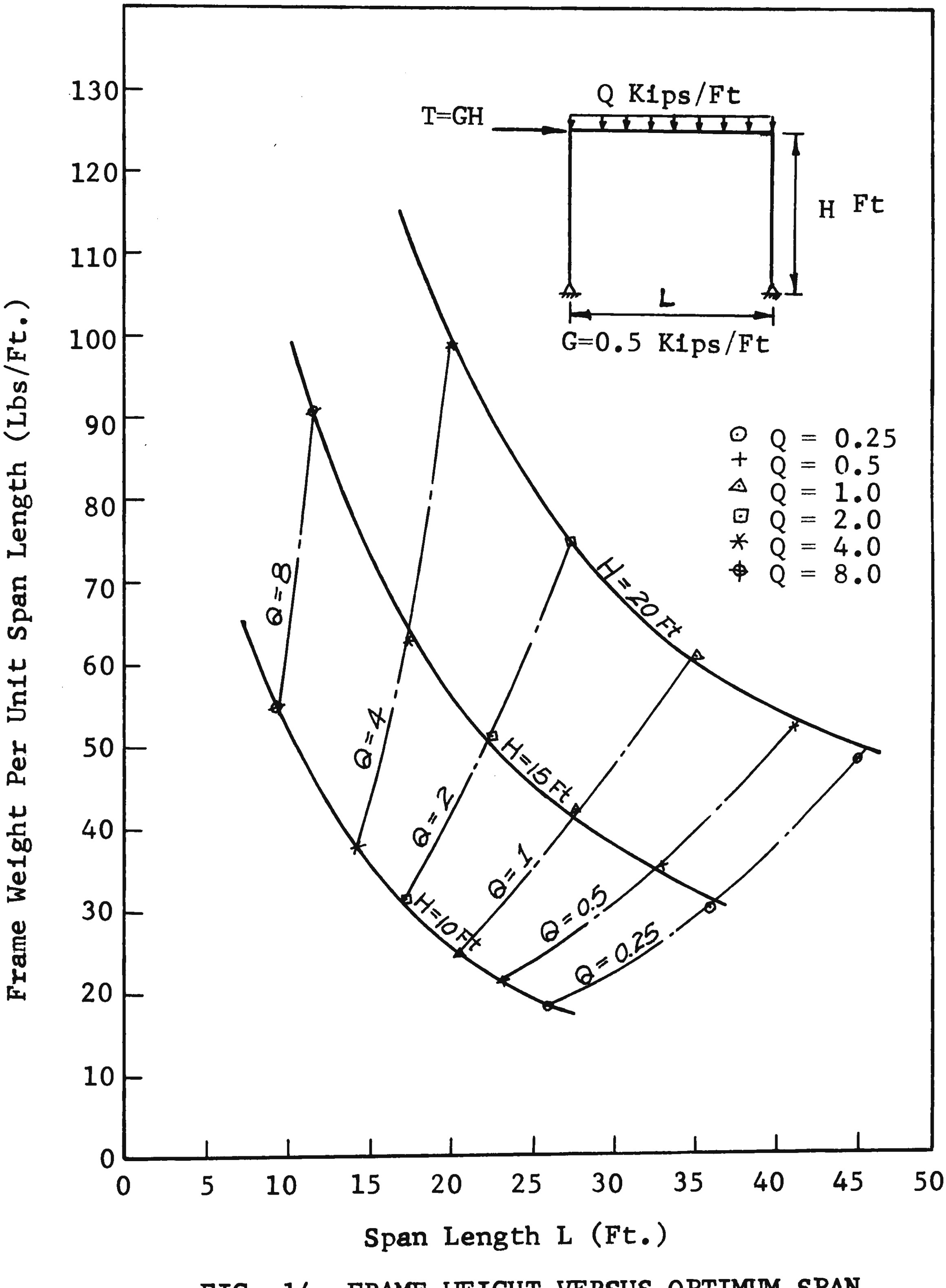


FIG. 14 FRAME WEIGHT VERSUS OPTIMUM SPAN LENGTH FOR HINGED END PORTAL FRAMES WITH WIND LOAD OF G=0.50

V. DISCUSSION AND CONCLUSION

5.1 Discussion

Chart solutions are possible in simplifying the procedure for the solution of single-span frames. The virtual work equations can be expressed as formulas which would reflect both the frame geometry and the loading conditions. Alternatively curves may be prepared which present the solution in chart form. It enables the engineer to determine the required plastic moment of a single-span frame with the aid of charts in a fraction of the time required in a "routine" plastic analysis. Figs. 7 and 8 present solutions to a fixed end portal frame. Their use is indicated by the examples which are shown in Appendix A.

In Figs. 7 through 10, there appear to be several anomalies in the curves as plotted.

(1) Those anomalies in Figs. 7 and 8 can be explained by noting that there will be a discontinuity in the curve when a change in failure mechanism occurs. In Fig. 7, when Q = 8, the frame will fail by composite mechanism at L = 30 Ft. and beam mechanism at L = 35 Ft. Therefore, the curve with Q = 8 is discontinuous over that portion as shown by a dashed line.

(2) In Fig. 9, it should be noted that for some frames, especially those with low Q loadings, the frame weight per unit span length actually in-

creases for short span lengths. This is due to the fact that in this range minimum column size governs rather than column load; as a result, some frames will show an optimum span length for minimum frame weight per unit span length. For example, when Q = 0.25, the optimum span length is seen to be 25 Ft. From the finite number of standard economic sections for design, when the span length is smaller than 25 Ft., the column section remains very nearly the same; so the frame weight per unit span length decreases as the span length increases to optimum span length.

(3) In Fig. 10, when the load is light and the span length is short, the increase of shearing forces due to uniform loading is less than the decrease of shearing forces due to the plastic moments developed at the ends of the beam. From this it can be seen that this results in an optimum span length for minimum axial load.

5.2 Conclusions

The method of optimization developed in this study allows for the determination of the minimum-weight design of steel frames within the restrictions imposed in Sec. 2.2. The method includes the effects of axial loading, overall frame instability due to sidesway, and the non-linear relationship between weight and moment capacity of standard sections.

Although standard sections are used in the frame computations for this study and frames using built-up sections can also be optimized provided that a linear weight-moment equation for the range of proposed built-up sections is determined.

Although gable and other non-orthogonal frames are not considered in the models, they can be optimized by the method proposed and accommodated by slightly modifying the computer program which was developed.

It is of course true that there are many factors which affect the cost of a structure besides its weight, and in a practical design, several different loading systems must often be considered. For example, subsequent to the selection of the individual members, problems of deflection, incremental collapse, cyclic loading, connections, clearance, etc., may need to be considered. Upon checking the adequacy of the minimum-weight design against these so-called secondary criteria, it may be found necessary to change one or more members. The computer program, however, provides a method of solution so rapid and automatic that it may be of value in giving the engineer a rough guide in the initial stages of his design work.

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VITA

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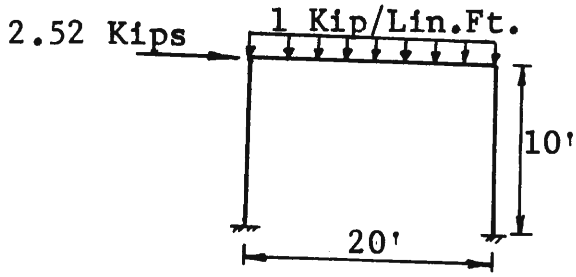
He received his secondary education at Taiwan Provincial Tainan First Middle School, Tainan, Taiwan, China. In September, 1956, he entered Taiwan Provincial Cheng Kung University as a freshman in Civil Engineering, and received his Bachelor of Science degree in Civil Engineering in July, 1960. After his graduation, he spent one year as a second lieutenant in the Chinese Air Force.

In January, 1965, he came to the United States and then enrolled as a graduate student at the University of Missouri at Rolla, for work toward his Master of Science degree in Civil Engineering.

APPENDIX A Illustrative Example

Example 1:



Given:

Roof load= 36 Lbs/sq Ft.
Wind load= 18 Lbs/sq Ft.
Frames are 20' on center
Load factor= 1.4

Design the frame for minimum weight

$$\text{Roof load} = \frac{36 \times 20 \times 1.4}{1000} = 1 \text{ Kip/Lin. Ft.}$$

$$\text{Wind force} = \frac{18 \times 20 \times 5 \times 1.4}{1000} = 2.52 \text{ Kips}$$

From Fig. 7 and Fig. 8 we get:

M_p (beam) = 39.33
M_p (column) = 14.85

Refer to Appendix B. The sections are selected as:

Beam - 12JR11.8
Column - 8JR6.5

The Critical Mechanism is Composite Mechanism.

Example 2:

Given:

Same as Example 1, except span length is 40 Ft.

From Fig. 7 and Fig. 8 we get:

M_p (beam) = 120.73
M_p (column) = 120.73

Refer to Appendix B. The sections are selected as:

Beam - 16B26
Column - 16B26

The Critical Mechanism is Beam Mechanism.

APPENDIX B Properties of Economic Sections (A-7 Steel)

No.	Nominal Depth	Weight Per Ft	Mp	Area	γ_x	Shape
1	6.00	4.40	7.70	1.30	2.37	6JR4.4
2	7.00	5.50	11.00	1.61	2.74	7JR5.5
3	8.00	6.50	14.85	1.92	3.12	8JR6.5
4	10.00	9.00	25.30	2.64	3.85	10JR9
5	12.00	11.80	39.33	3.45	4.57	12JR11.8
6	10.00	15.00	44.00	4.40	3.95	10B15
7	12.00	16.50	56.65	4.86	4.65	12B16.5
8	14.00	17.20	67.93	5.05	5.40	14B17.2
9	12.00	19.00	68.20	5.62	4.81	12B19
10	14.00	22.00	90.75	6.47	5.52	14B22
11	16.00	26.00	120.73	7.65	6.24	16B26
12	14.00	30.00	129.53	8.81	5.73	14WF30
13	14.00	34.00	149.88	10.00	5.83	14WF34
14	16.00	36.00	175.73	10.59	6.49	16WF36
15	16.00	40.00	199.30	11.77	6.62	16WF40
16	18.00	45.00	246.40	13.24	7.30	18WF45
17	18.00	50.00	277.20	14.71	7.38	18WF50
18	21.00	55.00	344.85	16.18	8.40	21WF55
19	21.00	62.00	396.28	18.23	8.53	21WF62
20	24.00	68.00	482.63	20.00	9.53	24WF68
21	24.00	76.00	550.28	22.37	9.68	24WF76
22	27.00	84.00	668.80	24.71	10.69	27WF84
23	27.00	94.00	763.68	27.65	10.87	27WF94
24	30.00	99.00	858.00	29.11	11.70	30WF99
25	30.00	108.00	950.13	31.77	11.85	30WF108
26	30.00	116.00	1038.40	34.13	12.00	30WF116
27	33.00	118.00	1139.33	34.71	13.02	33WF118
28	33.00	130.00	1281.50	38.26	13.23	33WF130
29	33.00	141.00	1411.30	41.51	13.39	33WF141
30	36.00	150.00	1594.45	44.16	14.29	36WF150
31	36.00	160.00	1714.08	47.09	14.38	36WF160
32	36.00	170.00	1833.43	49.98	14.47	36WF170
33	36.00	182.00	1971.48	53.54	14.52	36WF182
34	36.00	194.00	2109.80	57.11	14.56	36WF194
35	36.00	230.00	2592.43	67.73	14.88	36WF230
36	36.00	245.00	2772.00	72.03	14.95	36WF245
37	36.00	260.00	2959.00	76.56	15.00	36WF260
38	36.00	280.00	3209.25	82.32	15.12	36WF280
39	36.00	300.00	3451.25	88.17	15.17	36WF300

APPENDIX C FRAME WEIGHT VERSUS SPAN LENGTH

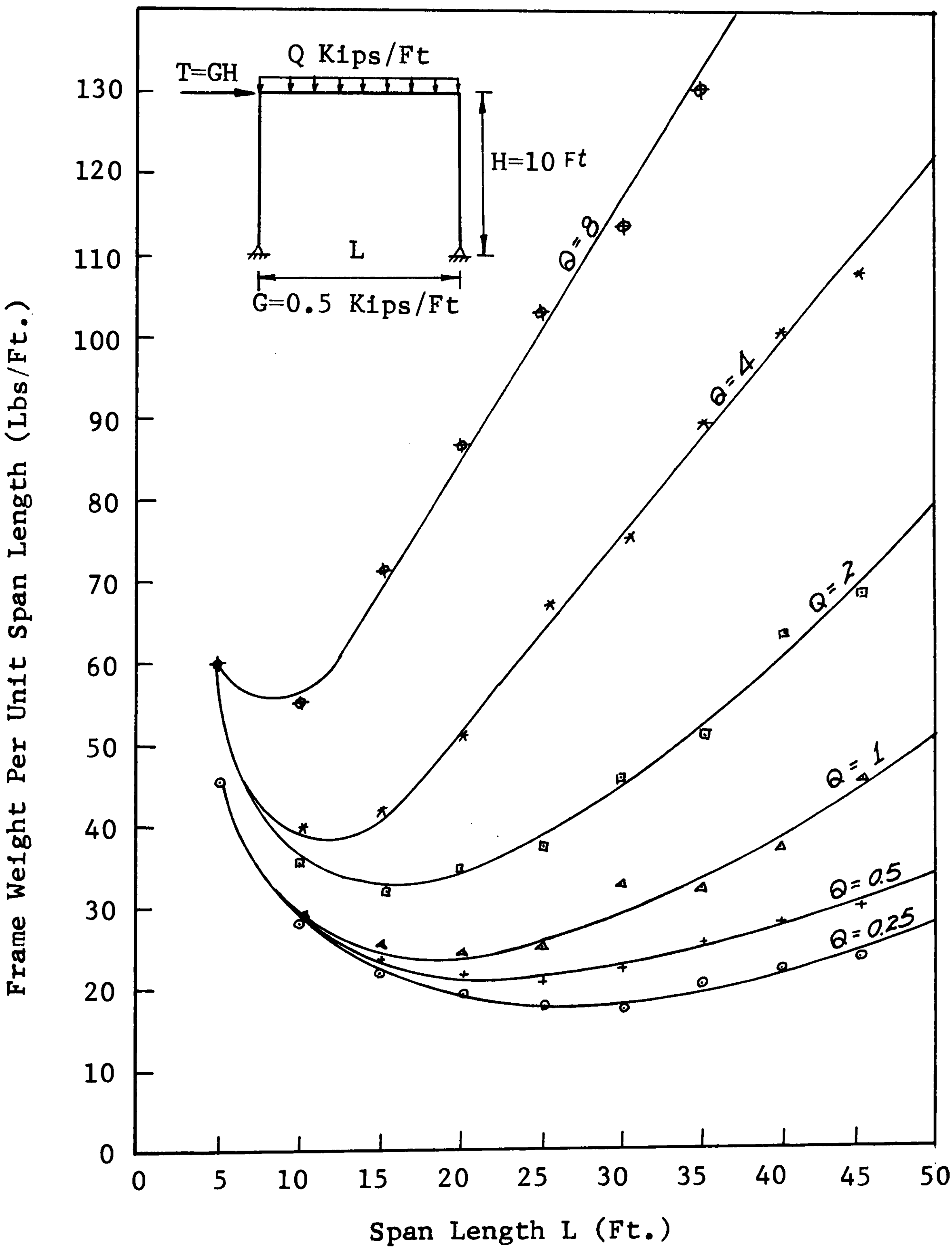


FIG. C-1 FRAME WEIGHT VERSUS SPAN LENGTH FOR A PIN-BASED PORTAL FRAME WITH H=10, G=0.5

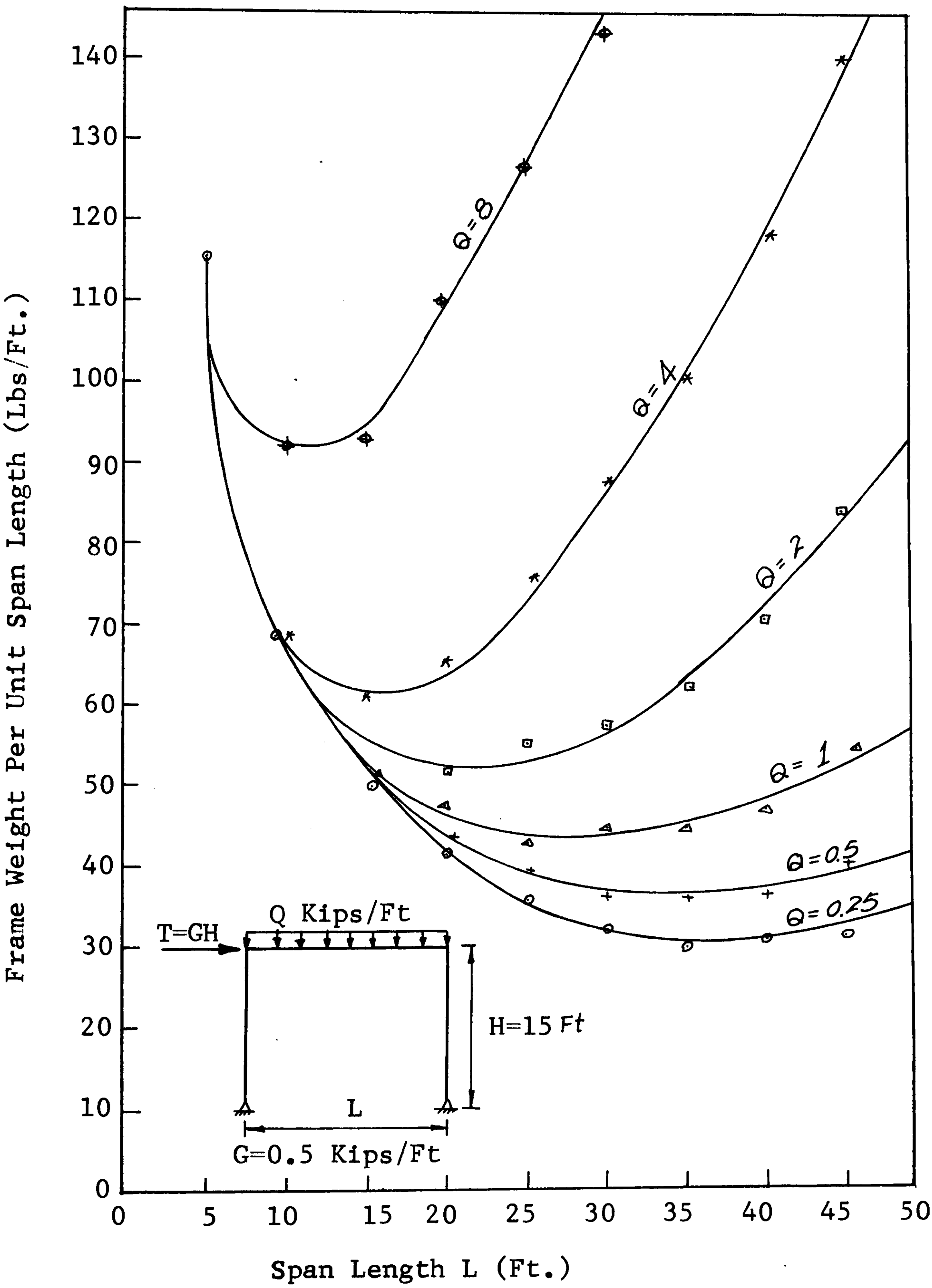


FIG. C-2 FRAME WEIGHT VERSUS SPAN LENGTH FOR A PIN-BASED PORTAL FRAME WITH H=15, G=0.5

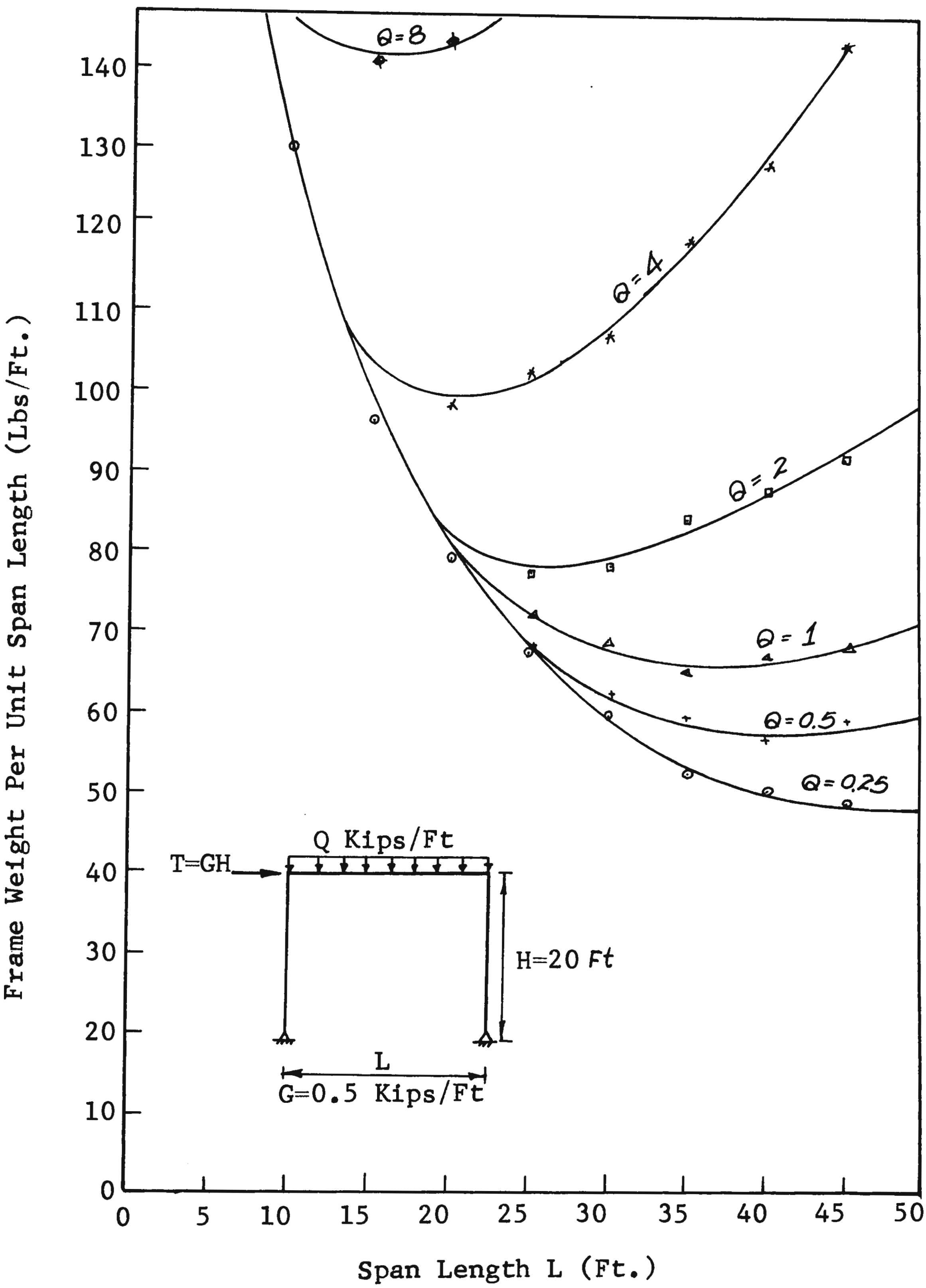


FIG. C-3 FRAME WEIGHT VERSUS SPAN LENGTH FOR A PIN-BASED PORTAL FRAME WITH H=20, G=0.5

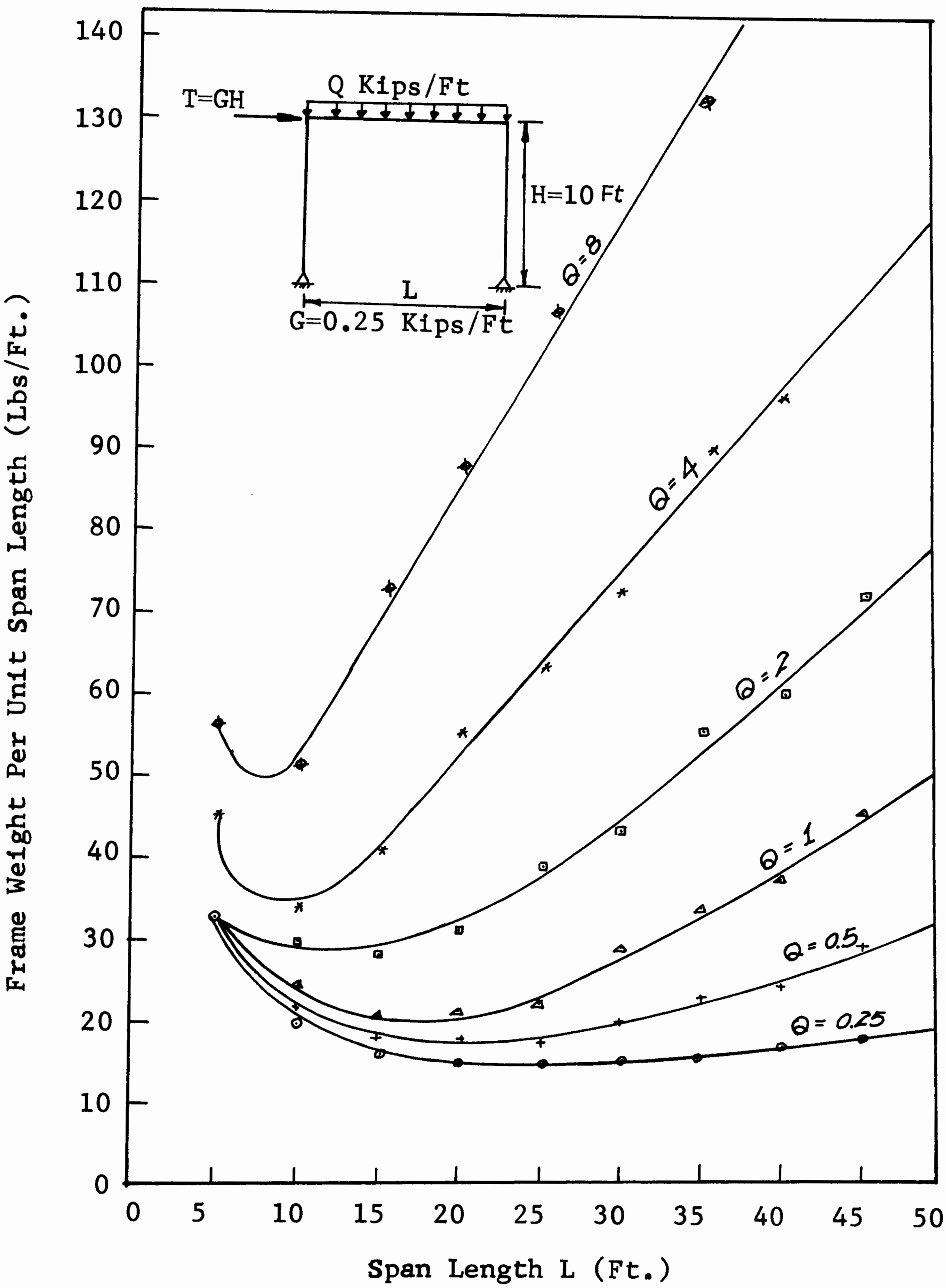


FIG. C-4 FRAME WEIGHT VERSUS SPAN LENGTH FOR A PIN-BASED PORTAL FRAME WITH H=10, G=0.25

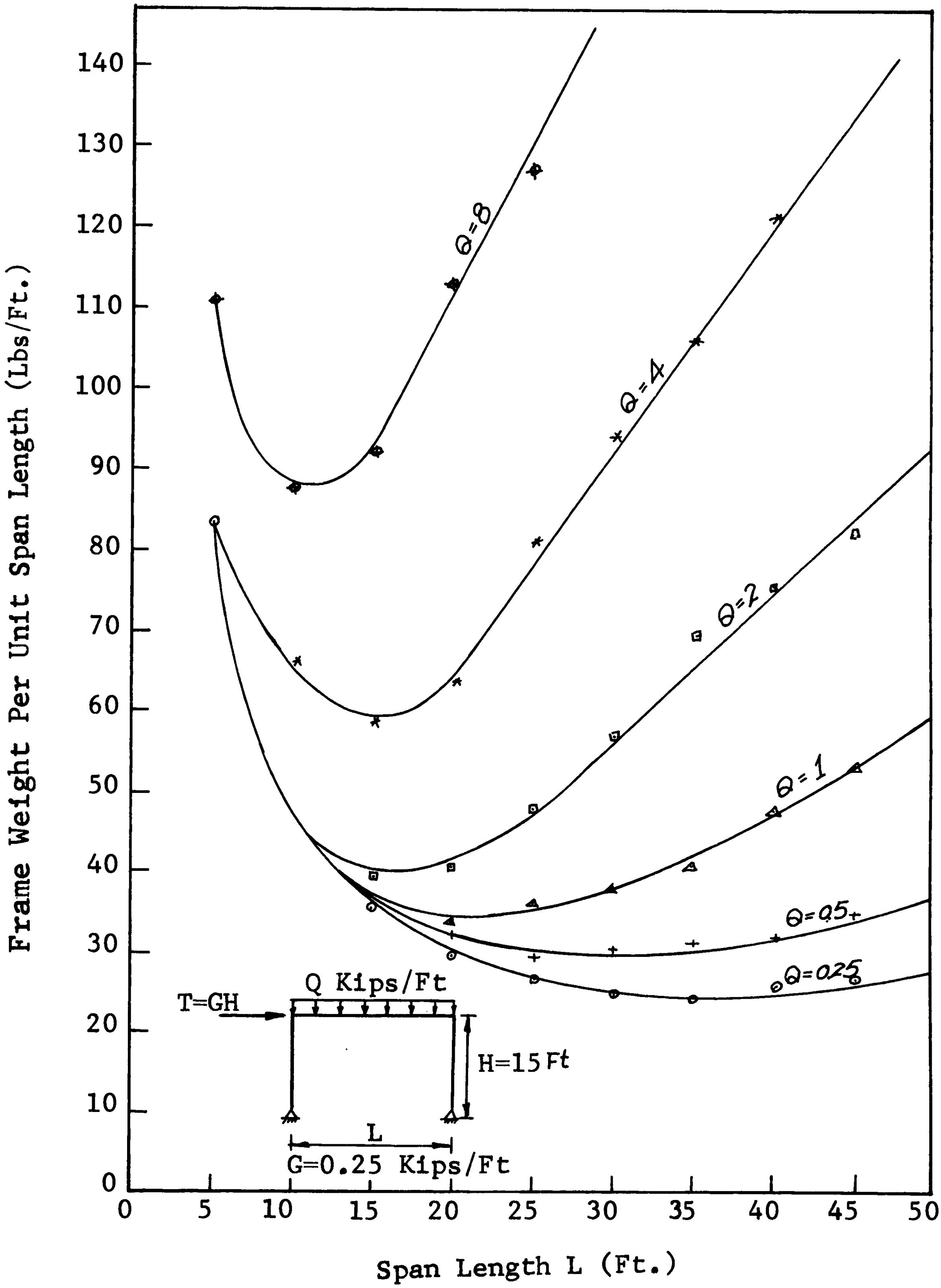


FIG. C-5 FRAME WEIGHT VERSUS SPAN LENGTH FOR A PIN-BASED PORTAL FRAME WITH H=15, G=0.25

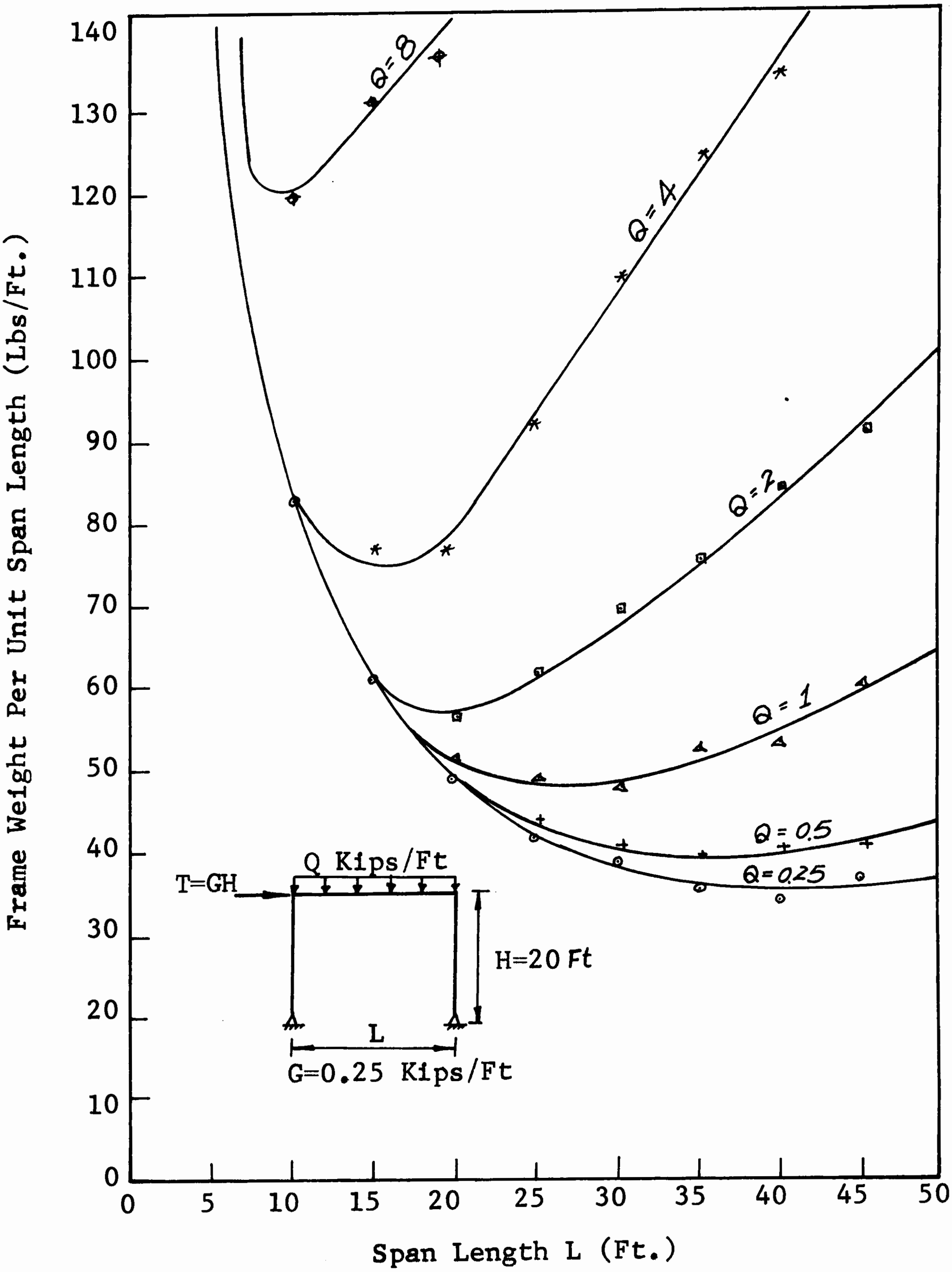


FIG. C-6 FRAME WEIGHT VERSUS SPAN LENGTH FOR A PIN-BASED PORTAL FRAME WITH H=20, G=0.25

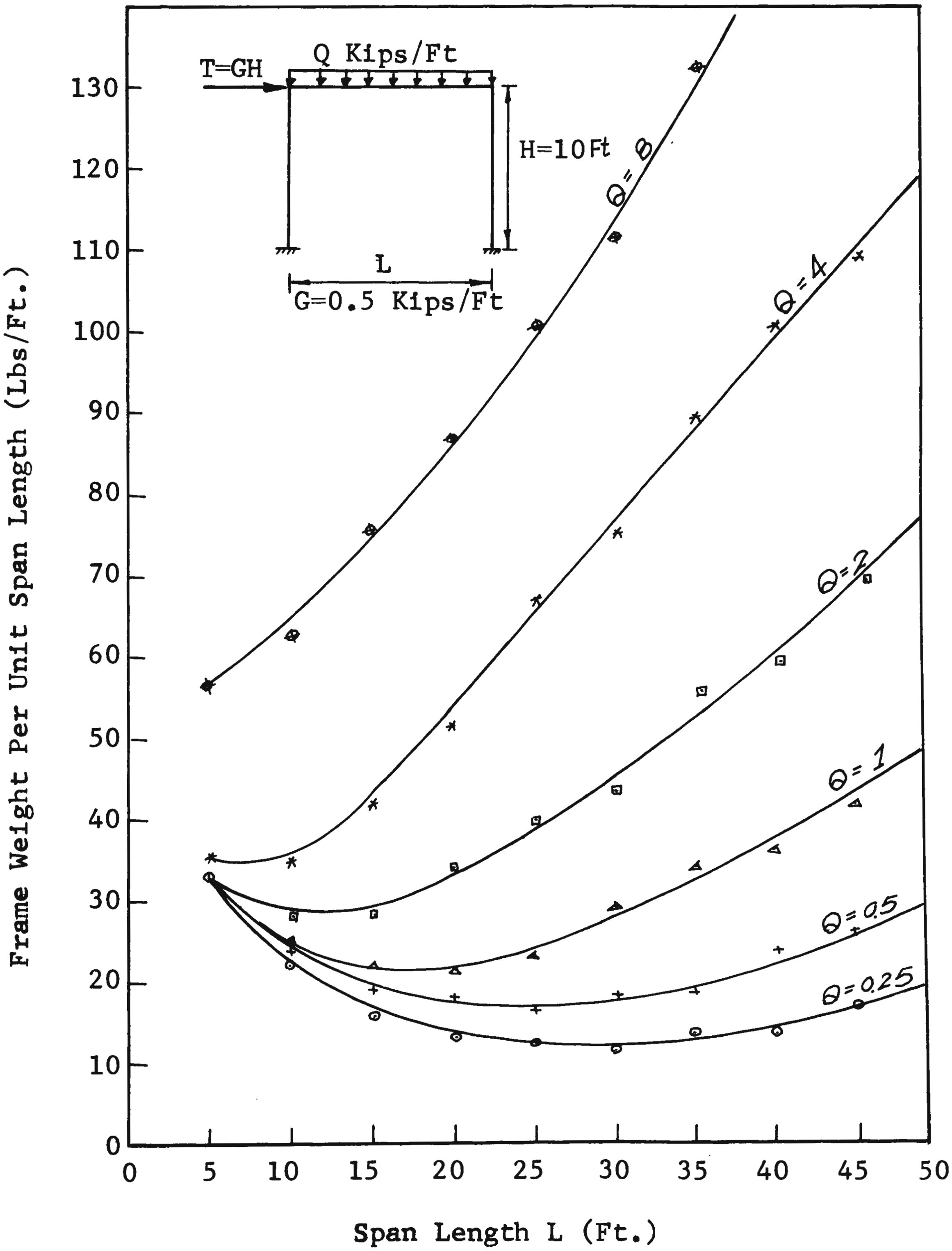


FIG. C-7 FRAME WEIGHT VERSUS SPAN LENGTH FOR A FIXED-BASED PORTAL FRAME WITH H=10, G=0.5

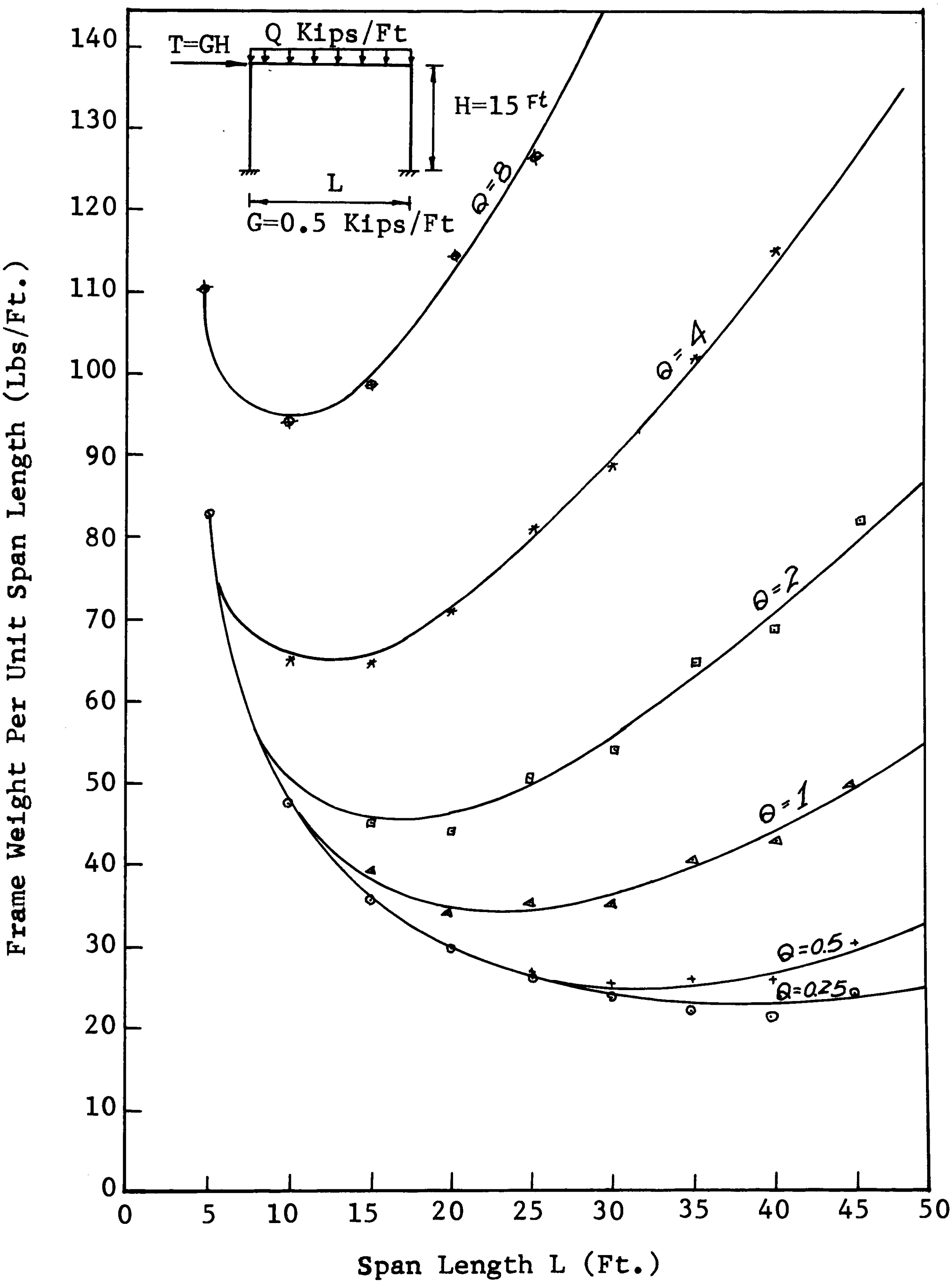


FIG. C-8 FRAME WEIGHT VERSUS SPAN LENGTH FOR A FIXED-BASED PORTAL FRAME WITH H=15, G=0.5

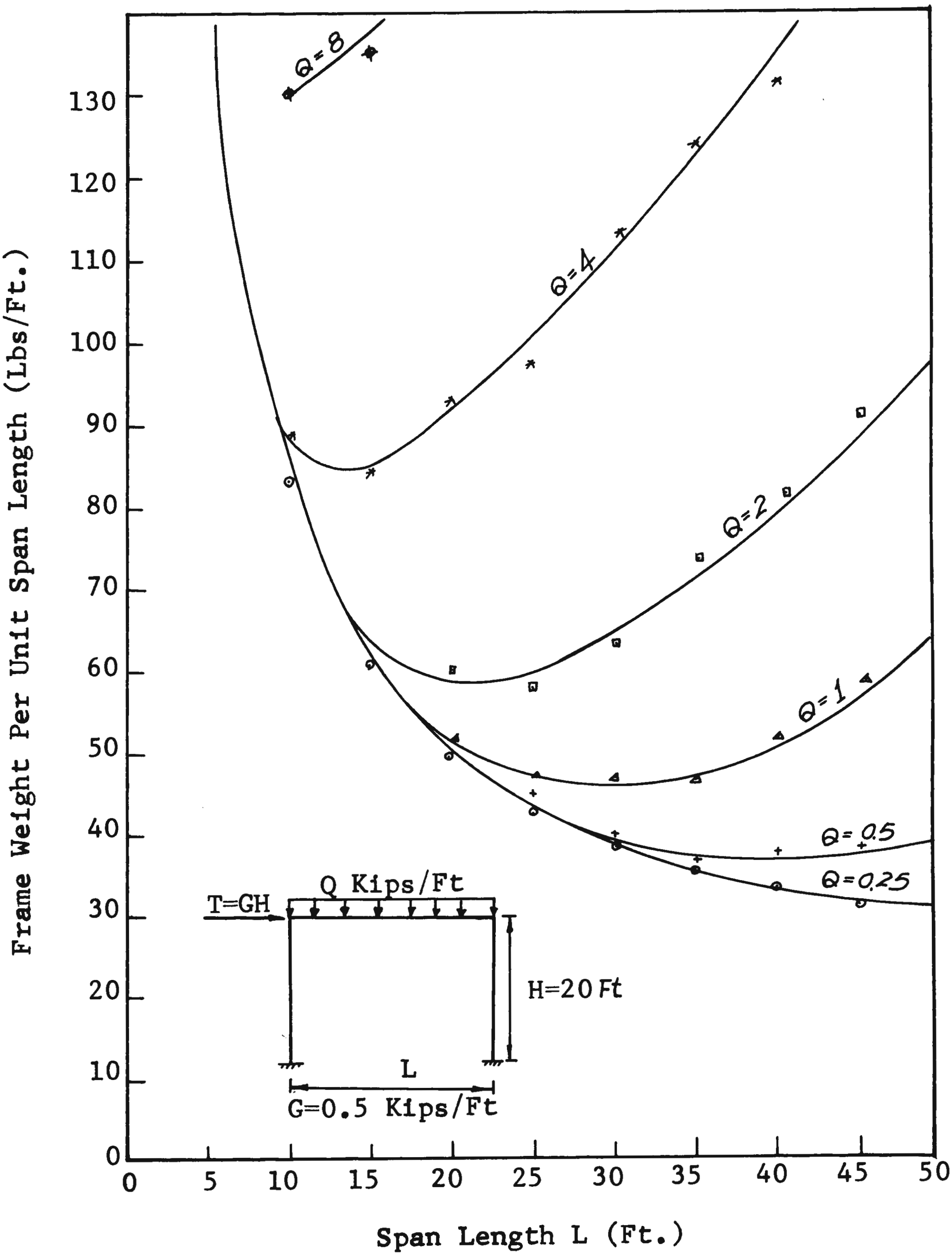


FIG. C-9 FRAME WEIGHT VERSUS SPAN LENGTH FOR A FIXED-BASED PORTAL FRAME WITH H=20, G=0.5

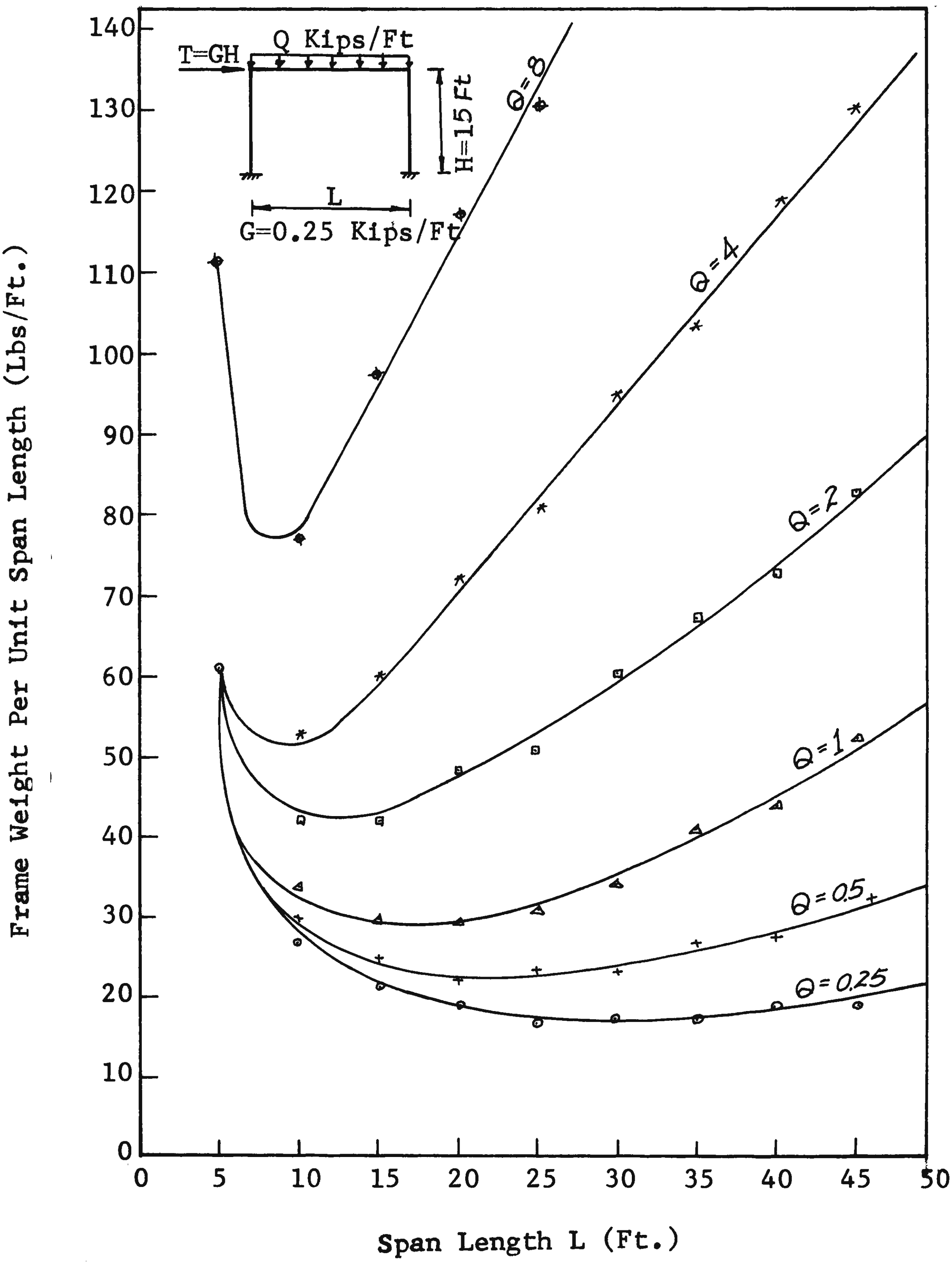


FIG. C-10 FRAME WEIGHT VERSUS SPAN LENGTH FOR A FIXED-BASED PORTAL FRAME WITH H=15, G=0.25

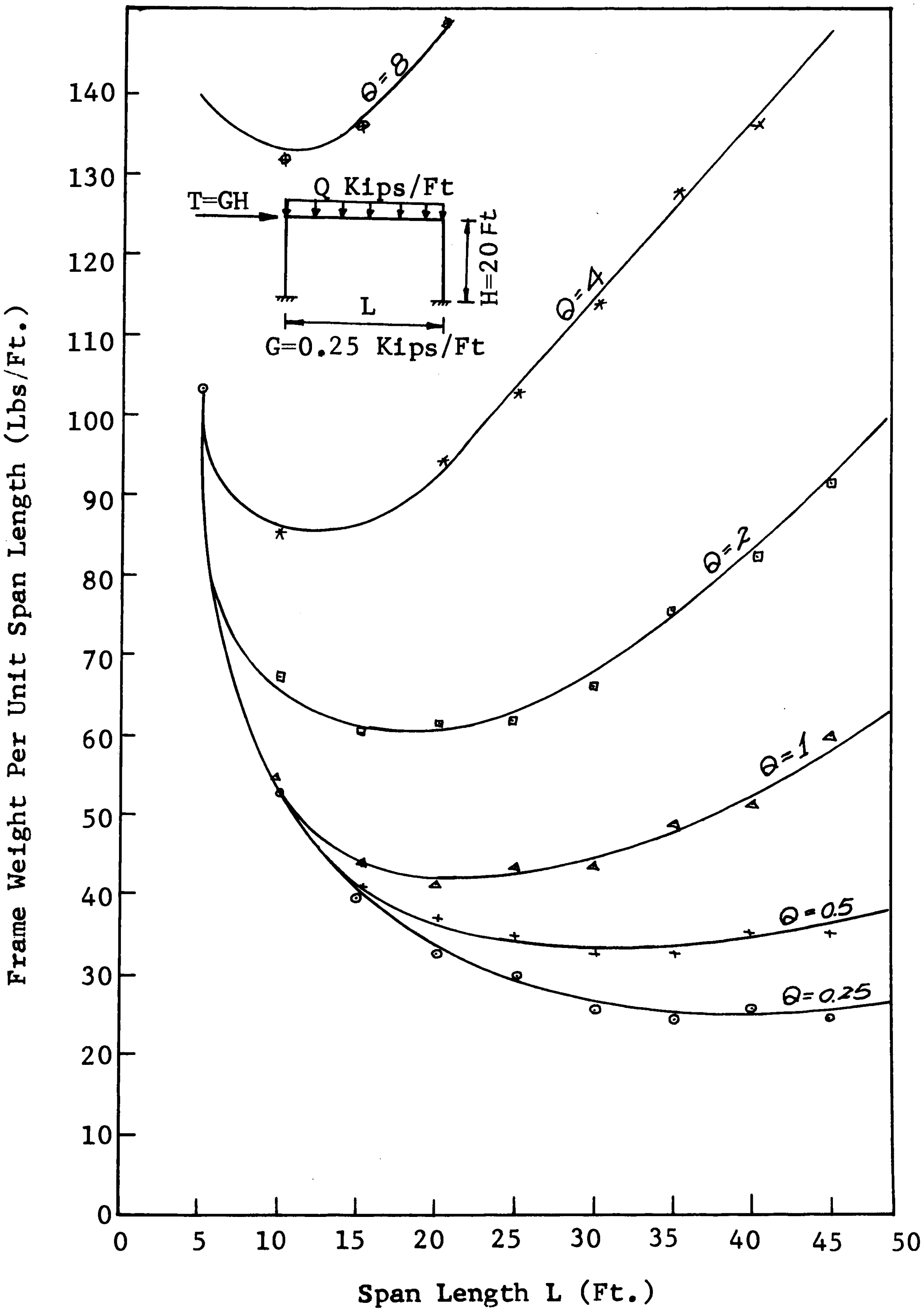


FIG. C-11 FRAME WEIGHT VERSUS SPAN LENGTH FOR A FIXED-BASED PORTAL FRAME WITH H=20, G=0.25

APPENDIX D

FLOW DIAGRAM FOR MINIMUM-WEIGHT PROGRAM

1 Start

↓

2 Generate Data

↓

3 Read:
(a) Section Properties
  1. Mp Capacity
  2. Area
  3. Rx
  4. Weight
(b) Mp range for beam and columns
(c) Yield stress of steel
(d) Problem number

↓

4 Write out all input values

↓

5 Calculate slope of best fit straight line by method of the least square for weight per foot vs. Mp equation for beam

↓

6 Calculate slope of best fit straight line by method of the least square for weight per foot vs. Mp equation for columns

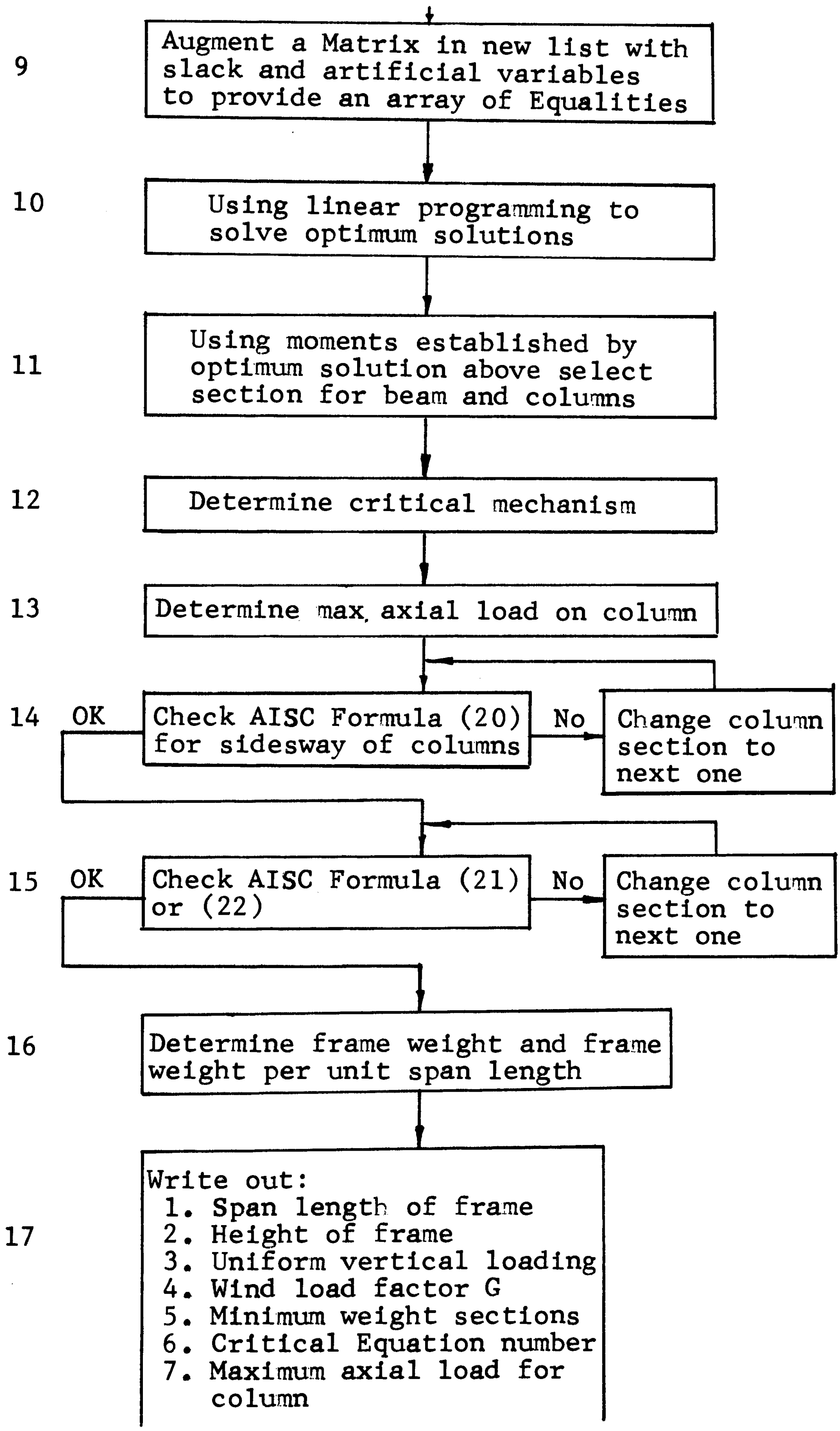
↓

7 Calculate objective function coefficients as products of member length and slopes of best fit straight lines

↓

8 Determine linear restrictions

↓


9
Augment a Matrix in new list with slack and artificial variables to provide an array of Equalities
10
Using linear programming to solve optimum solutions
11
Using moments established by optimum solution above select section for beam and columns
12
Determine critical mechanism
13
Determine max. axial load on column
14
OK
Check AISC Formula (20) for sidesway of columns
No
Change column section to next one
15
OK
Check AISC Formula (21) or (22)
No
Change column section to next one
16
Determine frame weight and frame weight per unit span length
17
Write out:
1. Span length of frame
2. Height of frame
3. Uniform vertical loading
4. Wind load factor G
5. Minimum weight sections
6. Critical Equation number
7. Maximum axial load for column

8. Total frame weight
9. Total frame weight per ft. span

18 Is this last problem to be solved? No → Go to Start

19 Stop

APPENDIX E COMPUTER PROGRAM FOR MINIMUM-WEIGHT DESIGN OF STEEL FRAMES

```
*LIST PRINTER
*ALL STATEMENT MAP
C   C***20035CEX024  I-CHEN HUNG            02/26/66 FORTRAN 2     0000 000 0
C       MAIN PROGRAM- MINIMUM WEIGHT DESIGN OF STEEL FRAMES
        DIMENSION Y(21,61),NVIB(20),C(61),A(39,4),CB(20),YYY(21,61)
        DIMENSION NNBB(20)
        JE=0
C       INPUT DATA
        READ 3003,((A(I,II),II=1,4),I=1,39)
        READ 3002,M,N,NOPT
        READ 3001,((YYY(I,J),J=1,22),I=1,10)
        READ 3002,(NNBB(I),I=1,10)
        READ 3001,(C(I),I=3,22)
  3001  FORMAT (7F10.2)
  3002  FORMAT (20I3)
  3003  FORMAT (4F10.2)
C       CALCULATE SPAN LENGTH,HEIGHT, WIND FACTOR,VERTICAL LOAD
        AG=0.
        DO 3010 IG=1,2
        AG=AG+0.25
        AH=5.
        DO 3010 IH=1,3
        AH=AH+5.
        AQ=0.125
        DO 3010 IQ=1,6
        AQ=AQ*2.
        AS=0.
        DO 3010 IS=1,12
        AS=AS+5.
        P=AQ*AS/4.
        T=AG*AH
        PRINT 3007
  3007  FORMAT (1H1)
        PRINT 3500,AS,AH,AG
  3500  FORMAT (1X,3HAS=,F5.0,2X,3HAH=,F5.0,2X,3HAG=,F7.3)
        PRINT 3600,AQ,P,T
```

```
3600 FORMAT (1X,3HAQ=,F7.2,2X,2HP=,F10.2,2X,2HT=,F10.2)
     VMPMI=AS**2*AQ/16.
     VMPMA=2.*VMPMI
     VMOMA=VMPMI
     K=0
     I=1
1004 IF(A(I,2)-VMPMA)1005,1003,1003
1003 BB=A(I,1)/A(I,2)
     AB=0.
     AC=AB
     BC=BB
     GO TO 1020
1005 I=I+1
     IF(A(I,2)-VMPMA)1006,1007,1007
1006 IF(I-39)1005,1030,1030
1030 PRINT 1031
1031 FORMAT (1X,26HALL SECTIONS ARE TOO SMALL)
     K=1
     GO TO 1100
1007 II=1
     IF(A(II,2)-VMPMI)1008,1009,1009
1009 BC=A(II,1)/A(II,2)
     AC=0.
     GO TO 1015
1008 II=II+1
     IF(A(II,2)-VMPMI)1008,1010,1010
1010 IJ=1
     WC=0.
     VMOW=0.
     VMO=0.
     VVMO=0.
     DO 1011 IJ=1,II
     VMO=VMO+A(IJ,2)
     VVMO=VVMO+A(IJ,2)**2
```

```
      WC=WC+A(IJ,1)
 1011 VMOW=VMOW+A(IJ,1)*A(IJ,2)
      SUMC=II
      DC=SUMC*VVMO-VMO**2
      AC=(WC*VVMO-VMO*VMOW)/DC
      BC=(SUMC*VMOW-WC*VMO)/DC
 1015 IK=II
      W=0.
      VMPW=0.
      VMP=0.
      VVMP=0.
      DO 1012 IK=II,I
      VMP=VMP+A(IK,2)
      VVMP=VVMP+A(IK,2)**2
      W=W+A(IK,1)
 1012 VMPW=VMPW+A(IK,1)*A(IK,2)
      SUM=I-II+1
      D=SUM*VVMP-VMP**2
      BB=(SUM*VMPW-W*VMP)/D
      AB=(W*VVMP-VMP*VMPW)/D
 1020 TT=T*AH
      PS=AS*P
      PRINT 1500,BB,AS,BC,AH
 1500 FORMAT (1X,3HBB=,F9.3,2X,3HAS=,F9.3,2X,3HBC=,F9.3,2X,3HAH=,F9.3)

      C(1)=-BB*AS
      C(2)=-BC*2.*AH
      YYY(1,23)=PS/4.
      YYY(2,23)=PS/2.
      YYY(3,23)=TT/2.
      YYY(4,23)=TT/4.
      YYY(5,23)=1.5*PS+3.*TT
      YYY(6,23)=YYY(5,23)
      YYY(7,23)=PS+TT
      YYY(8,23)=YYY(7,23)
```

```
      YYY(9,23)=1.5*PS+TT
      YYY(10,23)=YYY(9,23)
 1100 CONTINUE
      IF(K)3011,3011,3010
 3011 CONTINUE
C     LINEAR PROGRAMMING WITH THE SIMPLEX METHOD
C     UNIFORM LOADING IS REPLACED BY 5 CONCENTRATED LOADING
C NOPT=0 NO TABLEAUS PRINTED, =1 LAST ONLY, =2 ALL ARE PRINTED.
      DO 512 I=1,10
      NVIB(I)=NNBB(I)
      DO 512 J=1,23
  512 Y(I,J)=YYY(I,J)
      M1=M+1
      NVIB(M1)=0
      N1=N+1
C     Y(I,N1)=XB(I), THAT IS THE N1 COLUMN OF Y IS THE SOLUTION.
      C(N1)=0.
      DO 3 I=1,M
      NI=NVIB(I)
    3 CB(I)=C(NI)
      DO 4 J=1,N1
      Y(M1,J)=-C(J)
      DO 4 I=1,M
    4 Y(M1,J)=Y(M1,J)+CB(I)*Y(I,J)
      NTG=0
      IF(NOPT-2)6,7,6
    7 NT=0
      GO TO 300
    6 DO 8 J=1,N
      IF(Y(M1,J))9,8,8
    8 CONTINUE
      GO TO 100
    9 K=J
      IF(N-K)10,11,10
   10 K1=K+1
      DO 12 J=K1,N
```

```
      IF(Y(M1,J)-Y(M1,K))9,12,12
   12 CONTINUE
   11 DO 13 I=1,M
      IF(Y(I,K))13,13,14
   13 CONTINUE
      GO TO 200
   14 NR=I
      IF(NR-M)15,16,16
   15 NR1=NR+1
      FACT=Y(NR,N1)/Y(NR,K)
      DO 17 I=NR1,M
      IF(Y(I,K))17,17,18
   18 IF(Y(I,N1)/Y(I,K)-FACT)14,17,17
   17 CONTINUE
   16 YRK=Y(NR,K)
      NVIB(NR)=K
C     TRANSFORMATION EQUATIONS
      DO 19 J=1,N1
   19 Y(NR,J)=Y(NR,J)/YRK
      DO 20 I=1,M1
      IF(I-NR)21,20,21
   21 YIK=Y(I,K)
      DO 22 J=1,N1
   22 Y(I,J)=Y(I,J)-YIK*Y(NR,J)
   20 CONTINUE
      IF(NOPT-2)6,7,6
  100 IF(NOPT-1)23,24,23
   24 NT=1
      GO TO 300
   23 PRINT224,Y(M1,N1)
      PRINT 25
      PRINT 26
      X1=0.0
      X2=0.0
```

```
      DO 110 I=1,10
      IF(NVIB(I)-1)109,109,111
  109 X1=Y(I,N1)
      GO TO 110
  111 IF(NVIB(I)-2)110,112,110
  112 X2=Y(I,N1)
  110 CONTINUE
      DO 27 I=1,M
   27 PRINT 28,NVIB(I),Y(I,N1)
      GO TO 34
  300 PRINT 29
      DO 31 I=1,M1
   31 PRINT 32,NVIB(I),Y(I,N1),(Y(I,J),J=1,N)
      IF(NT)34,66,34
   66 CONTINUE
      GO TO 6
   34 CONTINUE
      GO TO 39
  200 PRINT 42,K
      NT=-1
      IF(NOPT-1)34,300,34
   39 CONTINUE
   28 FORMAT(I8,E18.8)
   42 FORMAT(48H THE OBJECTIVE FUNCTION IS NOT BOUNDED ABOVE. K=,I4)
  224 FORMAT(27H THE MAXIMUM VALUE OF Z IS ,E18.8)
   25 FORMAT(25H OPTIMAL SOLUTION FOLLOWS)
   26 FORMAT(16H VARIABLE  VALUE)
   29 FORMAT(51H VIB        XB     A1      A2      A3      A4      A5    )
   32 FORMAT(I3,F10.2,10F7.1)
C     SUBROUTINE CE24PM
      MB=0
 2001 MB=MB+1
      IF(A(MB,2)-X1)2001,2002,2002
 2002 NC=0
```

```
2003 NC=NC+1
     IF(A(NC,2)-X2)2003,2004,2004
2004 PRINT 2024
2024 FORMAT (1X,14HLEFT HAND SIDE,5X,15HRIGHT HAND SIDE,4X,13HCRITICAL
    1MECH,3X,10HAXIAL LOAD)
     RP=0.
     DO 2030 IJ=1,10
     DL=YYY(IJ,1)*X1+YYY(IJ,2)*X2
     IF(DL-YYY(IJ,23))2007,2007,2008
2008 JJ=0
     R=0.
     GO TO 2030
2007 JJ=1
     GO TO (2011,2012,2013,2014,2015,2016,2017,2018,2019,2020),IJ
2011 R=P+4.*X1/AS
     GO TO 2021
2012 R=P+(2.*X1+2.*X2)/AS
     GO TO 2021
2013 R=2.*P+2.*X1/AS
     GO TO 2021
2014 R=2.*P+2.*X2/AS
     GO TO 2021
2015 R=1.5*P+2.667*X1/AS
     GO TO 2021

2016 R=1.5*P+1.3333*(X1+X2)/AS
     GO TO 2021
2017 R=P+4.*X1/AS
     GO TO 2021
2018 R=P+2.*(X1+X2)/AS
     GO TO 2021
2019 R=0.5*P+8.*X1/AS
     GO TO 2021
2020 R=0.5*P+4.*(X1+X2)/AS
2021 IF(RP-R)2022,2030,2030
```

```
 2022 RP=R
 2030 PRINT 2025,DL,YYY(IJ,23),JJ,R
 2025 FORMAT (1X,E15.8,5X,E15.8,11X,I2,7X,F10.3)
      PRINT 2026,RP
 2026 FORMAT (1X,25HMAX. AXIAL LOAD (KIPS) = ,F10.3)
 2033 SLR=12.0*AH/A(NC,4)
      IF(SLR-120.)2031,2031,2032
 2032 PRINT 2040
 2040 FORMAT (1X,48HSLENDERNESS RATIO EXCEEDS 120 SELECT NEW SECTION)
      NC=NC+1
      GO TO 2033
 2031 RATIO=RP/(33.*A(NC,3))
      FOM20=2.*RATIO+SLR/70.
      IF(FOM20-1.0)2035,2035,2034
 2034 PRINT 2041
 2041 FORMAT (1X,39HFORMULA 20 EXCEEDS 1 SELECT NEW SECTION)
      NC=NC+1
      GO TO 2033
 2035 IF(RATIO-0.15)2036,2036,2044
 2044 FOM21=(1.18-1.18*RATIO)*A(NC,2)
      IF(FOM21-X2)2037,2036,2036
 2037 PRINT 2038
 2038 FORMAT (1X,48HX2 EXCEEDS MO OF FORMULA (21),SELECT NEW SECTION)
      NC=NC+1
      GO TO 2033
 2036 CONTINUE
C     CALCULATE WEIGHT OF FRAME
      WB=AS*A(MB,1)
      WC=AH*A(NC,1)
      WT=WB+2.*WC
      WS=WT/AS
      JE=JE+1
      PRINT 3015
      PRINT 3004
      PRINT 3005,JE,AS,AH,AQ,AG,WS,A(MB,2),A(NC,2)
      PRINT 3006
```

```
      PRINT 3008,T,P,X1,X2,RP,WC,WB,WT
 3010 CONTINUE
      CALL EXIT
 3004 FORMAT (1X,71HPROB.NO. SPAN  HEIGHT Q LOAD  G FACTOR WT/UNIT SPAN
     1MOMENT B  MOMENT C )
 3005 FORMAT (2X,I3,F8.0,F7.0,F7.2,F9.2,F12.2,2F11.2)
 3006 FORMAT (1X,69HWIND LOD P LOAD THOR. BEAM  THOR.COL. MAX.AX WT.COL
     1WT.BEAM TOTAL WT.)
 3008 FORMAT (1X,F8.2,F7.0,F12.2,F10.2,F7.0,F8.0,2F9.0)
 3015 FORMAT (1X,25HFIXED BASED PORTAL FRAMES)
      END
```

A PARTIAL LIST OF FORTRAN SYMBOLS

A = Properties of economic sections.
M = Number of rows in the Augment Matrix.
N = Number of columns in the Augment Matrix.
YYY = Coefficients by rows in the Augment Matrix.
NNBB = Read in vectors in basis for first table.
C = Read in cost coefficients.
AG = Wind load factor.
AH = Height of frame.
AQ = Uniform loading on frame
AS = Span length of frame.
VMPMI = Minimum plastic moment for beam.
VMPMA = Maximum plastic moment for beam.
VMOMI = Minimum plastic moment for column.
VMOMA = Maximum plastic moment for column.
BB = Slope of best fit straight line for weight per foot vs. Mp equation for beam.
BC = Slope of best fit straight line for weight per foot vs. Mp equation for column.
X1 = Theoretical plastic moment for beam.
X2 = Theoretical plastic moment for column.
R = Axial load in the column.
SLR = Slenderness ratio.
WB = Weight of beam.
WC = Weight of column.
WT = Total weight of the frame.
WS = Frame weight per unit span length.

APPENDIX F ILLUSTRATION OF OUTPUT DATA

```
AS=  20.   AH=  10.   AG=    .250
AQ=     .50  P=        2.50   T=       2.50
BB=      .239   AS=    20.000   BC=      .292   AH=   10.000
THE MAXIMUM VALUE OF Z IS      -.13300198E+03
OPTIMAL SOLUTION FOLLOWS
VARIABLE  VALUE
      1      .12500000E+02
      8      .25000005E+02
      2      .12500000E+02
      7      .25000003E+02
     11      .25000001E+02
     12      .25000002E+02
      3      .18333336E-06
      4      .21666666E-06
      5      .12499999E+02
      6      .62500002E+01
LEFT HAND SIDE       RIGHT HAND SIDE     CRITICAL MECH    AXIAL LOAD
  .12500000E+02       .12500000E+02            1             5.000
  .25000000E+02       .25000000E+02            1             5.000
  .25000000E+02       .12500000E+02            0             0.000
  .12500000E+02       .62500000E+01            0             0.000
  .17500000E+03       .15000000E+03            0             0.000
  .17500000E+03       .15000000E+03            0             0.000
  .75000000E+02       .75000000E+02            1             5.000
  .75000000E+02       .75000000E+02            1             5.000
  .12500000E+03       .10000000E+03            0             0.000
  .12500000E+03       .10000000E+03            0             0.000
MAX. AXIAL LOAD (KIPS) =        5.000
FIXED BASED PORTAL FRAMES
PROB.NO. SPAN  HEIGHT Q LOAD  G FACTOR WT/UNIT SPAN MOMENT B  MOMENT C
  16      20.     10.    .50      .25        13.00       14.85       14.85
WIND LOD P LOAD THOR. BEAM  THOR.COL. MAX.AX WT.COL WT.BEAM TOTAL WT.
    2.50      2.          12.50       12.50      5.      65.      130.      260.
```